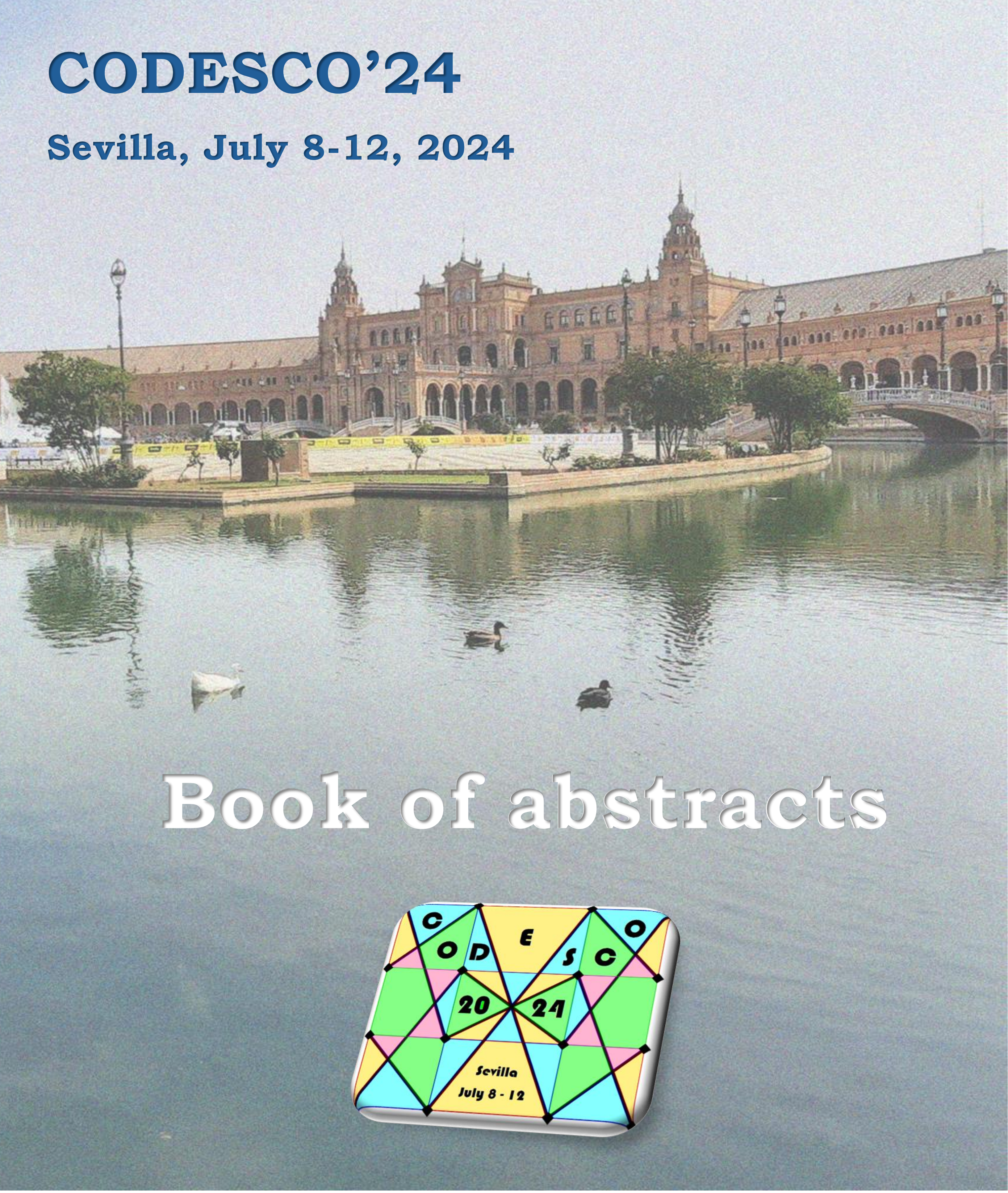
CODESCO'24

Sevilla, July 8-12, 2024



Book of abstracts

CODESCO 2024 Sevilla July 8 - 12

9th EUROPEAN CONGRESS OF MATHEMATICS
ECM Sevilla2024 July 15th to 19th

EMS EUROPEAN MATHEMATICAL SOCIETY

UNIVERSIDAD Ð SEVILLA 1505

departamento de matemática aplicada I

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Editor: Raúl M. Falcón.

All the abstracts have been prepared by the authors.

Website: `https://gestioneventos.us.es/codesco24`

Contents

Welcome

It is our pleasure to welcome you to Seville for **CODESCO’24**! Seville is the capital city of Andalusia, multicultural land of painters, writers and artists for hundred of years, and birthplace of flamenco and tapas. We sincerely wish that you all enjoy our beloved and beautiful city, even with its warm July weather. A little secret: We are sure that you will do it, because one of our most popular legends says that Seville would not abandon your hearts. This fact is constantly reminded when walking through the city, where you will see everywhere the word *“No8Do”*.

Here, the symbol ∞ does not only represent the infinity, but a wool skein, which means *madeja* in Spanish. Thus, the word *“No8Do”* constitutes indeed a hieroglyph that means *”No madeja Do”*, phonetic expression of *“No me ha dejado”*. In English: *“She has not abandoned me”*. If you give it a chance, Seville would not abandon your hearts.

***

CODESCO’24 is the second edition of the conference ***Combinatorial Designs and Codes***, whose first edition was held in Rijeka (Croatia) in 2021. The goal of the conference is to bring together researchers interested in combinatorial design theory, coding theory, graph theory, algebraic combinatorics and finite geometry, with particular emphasis on establishing new synergies among them, and new applications to other fields and to the real world, including artificial intelligence, communication networks, cryptography and machine learning.

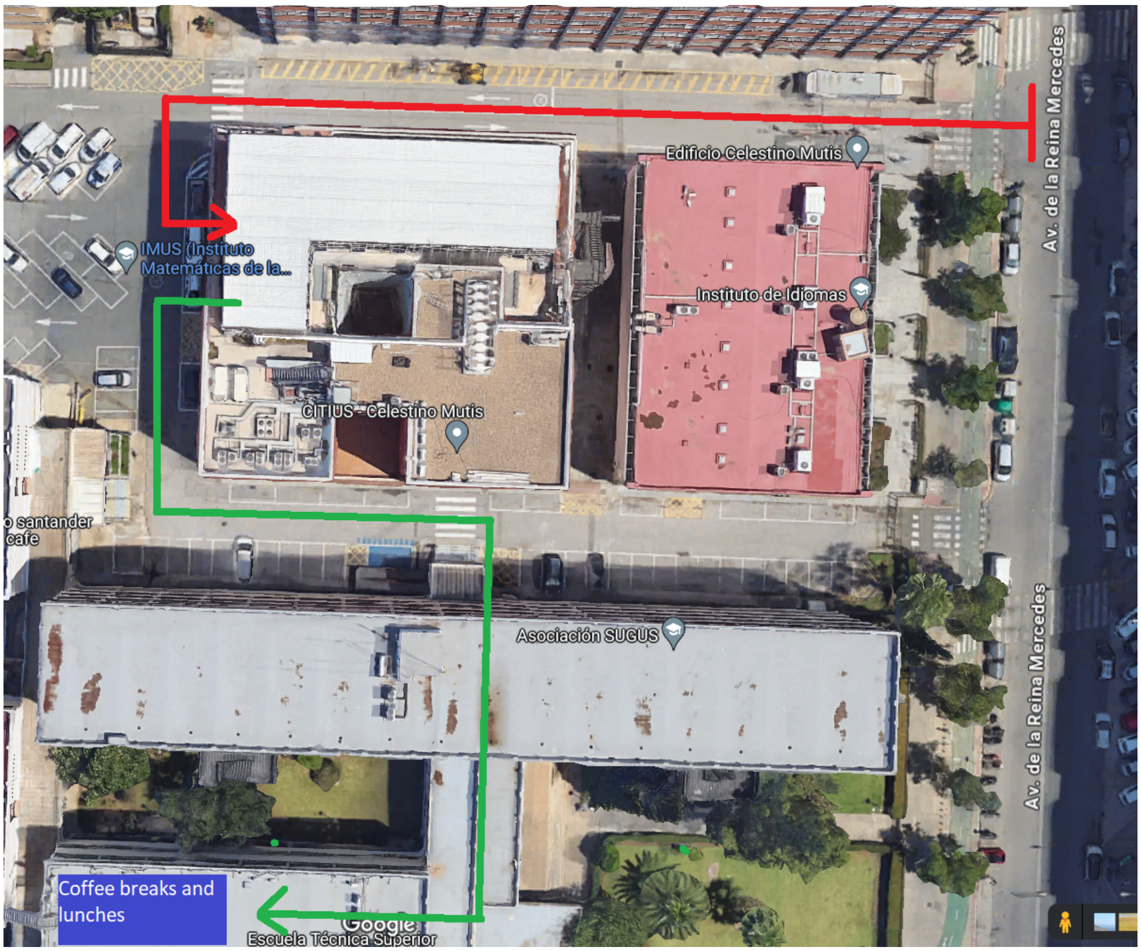
This second edition takes place from July 8 to 12, 2024, at the *Institute of Mathematics of the University of Seville* (IMUS). It is indeed a satellite event of the *9th European Congress of Mathematics*, which will also be held in Seville from July 15 to 19, 2024. In CODESCO’24, we have seven invited talks, one popular invited talk, and 41 contributed talks accepted by the Scientific Committee. In total, 58 participants from 16 different countries around the world.

Both the Organizing and the Scientific Committees thank all the participants for their interest and contribution to make this conference an important scientific event.

Venue

The conference takes place at the ***Institute of Mathematics of the University of Seville*** (***IMUS***). It is located on the *Reina Mercedes Campus*, in the southern part of the city. More precisely, the address is:

Edificio Celestino Mutis - CITIUS II, Av. de la Reina Mercedes, s/n, 41012 Sevilla.



The IMUS aims are to organize and develop research activities in all fields and aspects of mathematics and its applications, to promote qualitatively and quantitatively this research, to support the research groups in mathematics of the University of Seville and to foster collaboration between them, with other national and international research groups, in particular by promoting the interdisciplinarity, and with scientific, technological, health, and financial sectors, among others, which could ask for help from mathematics.

Conference programme

Monday, July 8

8:00 - 8:45	REGISTRATION
8:45 - 9:00	OPENING
9:00 - 9:55	[Session chair: Raúl Falcón] **Ian Wanless** **INVITED TALK**: *Subsquares of Latin squares*
9:55 - 10:20	Michael Kinyon *Loops with squares in two nuclei*
10:20 - 10:50	COFFEE
10:50 - 11:15	[Session chair: Ian Wanless] Akihiro Yamamura *Graphs of Latin Regular Hexahedra*
11:15 - 11:40	Daniel Kotlar *On Latin Young Diagrams*
11:40 - 12:05	Lorenzo Mella *On the Hadamard multiary quasigroup product*
12:05 - 12:30	Manuel González-Regadera *Critical sets based on non-trivial autoparatopisms of Latin squares*
12:30 - 12:55	Nour Alnajjarine *Linear Systems of Conics over Finite Fields*
13:00 - 14:30	LUNCH
14:30 - 15:25	[Session chair: Félix Gudiel] **Jaime Gutiérrez Gutiérrez** **INVITED TALK**: *Local permutation polynomials and Latin hypercubes*
16:15 - 16:45	COFFEE
16:45 - 18:00	Guided walk to Royal Alcázar
18:00 - 20:00	INDIVIDUAL VISIT TO ROYAL ALCÁZAR
20:00 - ...	Exploring Santa Cruz neighbourhood (Tapas)

Tuesday, July 9

9:00 - 9:55	[Session chair: Marco Buratti] **Anita Pasotti** **INVITED TALK**: *Graphs with prescribed edge-lengths: open problems and new results*
9:55 - 10:20	Onur Agirseven *Grid-Based Graphs, Linear Realizations, and the Buratti-Horak-Rosa Conjecture*
10:20 - 10:50	COFFEE
10:50 - 11:15	[Session chair: Daniel Kotlar] Toru Hasunuma *Exponentiation of Graphs*
11:15 - 11:40	Manuel Ceballos *New advances on graph families associated with graphicable algebras*
11:40 - 12:05	Angela Aguglia *On regular sets of affine type in finite Desarguesian planes and related codes*
12:05 - 12:30	Sibel Özkan *Complete Solutions to the Uniform Hamilton-Waterloo Problem*
12:30 - 12:55	Fatih Yetgin *On Some Cases of the Directed Uniform Hamilton-Waterloo Problem*
13:00 - 14:30	LUNCH
14:30 - 15:25	[Session chair: Dean Crnković] **Andrea Švob** **INVITED TALK**: *Some constructions of strongly regular graphs and digraphs*
15:25 - 15:50	Vladislav Kabanov *Strongly regular graphs decomposable into divisible design graph and Delsarte clique*
15:50 - 16:15	Valentino Smaldore *Hemisystems and Strongly Regular Graphs*
16:15 - 16:45	COFFEE

19:00 - 20:30	*"SEVILLA EXPO 1929"* TOUR

Wednesday, July 10

9:00 - 9:55	[Session chair: Andrés Armario] **Patrick Solé** **INVITED TALK**: *Hadamard matrices and spherical designs*
9:55 - 10:20	Dean Crnković *Constructing doubly even self-dual codes from Hadamard matrices*
10:20 - 10:50	COFFEE
10:50 - 11:15	[Session chair: Jim Davis] Qing Xiang *Storage Codes on Triangle-Free Graphs*
11:15 - 11:40	Miguel Ángel Navarro Pérez *Linearly equivalent flag codes*
11:40 - 12:05	Alexander Gavrilyuk *A linear programming bound for sum-rank-metric codes*
12:05 - 12:30	Vedrana Mikulić Crnković *Regular digraphs and related linear codes*
12:30 - 12:55	Ivona Traunkar *LCD codes related to some combinatorial structures*
13:00 - 14:30	LUNCH
14:30 - 14:55	[Session chair: Patrick Solé] Miguel Sales Cabrera *Direct product group codes and derived quantum codes*
14:55 - 15:20	Domingo Gómez-Pérez *On the combinatorial properties of shrinking sequences*
15:25 - 15:50	Stefan Trandafir *Contextual configurations*
15:50 - 16:15	Cristian Camilo Meneses *Resolvable $\lambda-$Golomb rules and resolvable cyclic configurations*
16:15 - 16:45	COFFEE

18:00 - 20:00	[Session chair: Raúl Falcón] **José Miguel Díaz-Báñez** **INVITED POPULAR TALK**: *Mathematics and Flamenco: An unexpected partnership*

Thursday, July 11

9:00 - 9:55	[Session chair: Anita Pasotti] **Marco Buratti** **INVITED TALK**: *Additive combinatorial designs*
9:55 - 10:20	Francesca Merola *Additive Graph Decompositions*
10:20 - 10:50	COFFEE
10:50 - 11:15	[Session chair: Qing Xiang] Vedran Krčadinac *On mosaics of designs*
11:15 - 11:40	Jim Davis *New Spence Difference Sets*
11:40 - 12:05	Anamari Nakic *Hadamard Partitioned Difference Families*
12:05 - 12:30	Laura Marie Johnson *New cyclic PBIBD(2)s obtained using finite field cyclotomic*
12:35 - 12:55	Lucija Relić *On designs of degree* 3
13:00 - 14:30	LUNCH
14:30 - 14:55	[Session chair: Santiago Barrera] Sophie Toulouse *Alphabet reduction pairs of tables*
14:55 - 15:20	Giovanni Falcone *On the additive embedding of* $\mathrm{PG}(2,q)$ *into* $\mathrm{AG}(3,q)$
15:25 - 15:50	Valentina Pepe *On the maximum field of linearity of linear sets*
15:50 - 16:15	Stephen Humphries *Schur rings over* $\mathrm{Sp}(\mathbf{n},\mathbf{2})$ *and multiplicity one subgroups*
16:15 - 16:45	COFFEE
16:45 - 17:10	[Session chair: Andrés Armario] Pastora Revuelta *Determining exact values of 4-color-off-diagonal generalized Schur numbers*
17:10 - 17:35	Soukaïna Mahzoum *Skew-adjacency matrices of tournaments with bounded principal minors*

20:00 - 22:00	CONFERENCE DINNER (*Restaurant "El Cabildo"*)
22:00 - 23:30	*NIGHT CITY* TOUR
23:30	METROPOL PARASOL VISIT (*"The mushrooms"*)

Friday, July 12

	[Session chair: Víctor Álvarez]
9:35 - 10:20	**Ilias Kotsireas**
	INVITED TALK: *130+ years of the Hadamard conjecture*
10:20 - 10:50	COFFEE
	[Session chair: Víctor Álvarez]
10:50 - 11:15	Robert Craigen
	Dimension of affine classes
11:15 - 11:40	Santiago Barrera Acevedo
	Cocyclic Two-Circulant Core Hadamard Matrices
11:40 - 12:05	Ronan Egan
	Hadamard matrices in centraliser algebras of monomial representations
12:05 - 12:30	Inés Mora
	Some results on Graphic Topology defined on Tournaments
12:30 - 12:55	Juan Núñez-Valdés
	Algebraically-informed deep networks for evolution algebras
12:55 - 13:00	CLOSING
13:00 - 14:30	LUNCH

INVITED TALKS

Additive combinatorial designs

Thursday 9h00

Marco Buratti

Sapienza University of Rome, Italy

Abstract

I would like to speak about some developments and extensions of the theory of additive designs recently founded by A. Caggegi, G. Falcone and M. Pavone [**5**]. I will focus my attention especially on Heffter spaces [**4**] a new class of combinatorial designs generalizing the well known Heffter arrays [**6**].

References

[1] M. Buratti and F. Merola. Additive graph decompositions. In preparation.

[2] M. Buratti, and A. Nakic. Super-regular Steiner 2-designs. *Finite Fields Appl.* **85**: 102116, 2023.

[3] M. Buratti and A. Nakic, Additivity of symmetric and subspace designs. Preprint.

[4] M. Buratti, and A. Pasotti. Heffter spaces. Preprint.

[5] A. Caggegi, G. Falcone, and M. Pavone. On the additivity of block designs. *J. Algebr. Comb.* **45**: 271–294, 2017.

[6] A. Pasotti and J. H. Dinitz. A survey of Heffter Arrays. In: New Advances in Designs, Codes and Cryptography (editors: C. J. Colbourn and J. H. Dinitz). *Fields Institute Communications* **86**: 353–392, NADCC, 2022.

Local permutation polynomials and Latin hypercubes

Monday
14h30

JAIME GUTIÉRREZ GUTIÉRREZ

UNIVERSIDAD DE CANTABRIA - FACULTY OF MATHEMATICS

(Joint work with Raúl M. Falcón and Jorge Jiménez-Urroz)

Abstract

There is a bijective map between n-dimensional Latin hypercube of order a prime power q and local permutation polynomial in n variables with coefficients the finite field F_q of degree smaller than q in each variable. In this talk, I will study how the algebraic variety described by the set of coefficients of these polynomials allows the establishment of new approaches to the problems of counting, enumerating and classifying Latin hypercubes. I will also analyse the set of orthogonal Latin hypercubes and it relation to an orthogonal system of polynomials.

130+ years of the Hadamard conjecture

Friday 9h30

ILIAS S. KOTSIREAS

UNIVERSITY - CARGO LAB, WILFRID LAURIER UNIVERSITY, WATERLOO, ON, CANADA

Abstract

In 1893, Jacques Salomon Hadamard showed that if H is a square matrix of order n, with real entries of absolute value ≤ 1, then it satisfies the determinant bound $|\det(H)| \leq n^{\frac{n}{2}}$. Matrices that satisfy the equality in this determinant bound became known as Hadamard matrices. Alternatively, Hadamard matrices are $n \times n$ matrices with elements exclusively taken from $\{-1, +1\}$, s.t. $H \cdot H^t = nI_n$, where the superscript t denotes matrix transposition and I_n denotes the $n \times n$ identity matrix. Beyond the trivial Hadamard matrices of orders $n = 1, 2$, there is a necessary existence condition on the order, namely that $n \equiv 0 \pmod 4$. The sufficiency of this existence condition is the celebrated Hadamard conjecture, namely that there exists a Hadamard matrix of order n, for every n which is a multiple of four. Despite the fact that a plethora of constructions for Hadamard matrices are available, their collective distilled power does not suffice to provide a positive resolution of the Hadamard conjecture. We shall survey a number of important results on the Hadamard conjecture since its inception. We will also describe a "structured" form of the Hadamard conjecture, based on the concept of Legendre pairs, introduced in 2001. The conjecture that Legendre pairs exist for every admissible (odd) order, implies the Hadamard conjecture. We shall survey a number of important results on Legendre pairs in the past 20+ years.

Graphs with prescribed edge-lengths: open problems and new results

Tuesday
9h00

Anita Pasotti

University of Brescia (Italy) - DICATAM

Abstract

The literature on graphs with prescribed edge-lengths is rich of attractive problems often linked with combinatorial topics of various kinds. A special attention has been devoted to a conjecture on Hamiltonian paths of the complete graph with a given list of edge-lengths, proposed by Buratti and then generalized in [**1**].

In [**4**] the authors proposed the following conjecture: a multiset L of n positive integers not exceeding n is the list of edge-lengths of a suitable near 1-factor F of the complete graph on $\{0, 1, \ldots, 2n\}$ if and only if it contains at most $\frac{2n+1-d}{2}$ multiples of any divisor d of $2n+1$. The case n prime was already considered in [**5**], where a complete, but non constructive, solution is presented. Also the case in which the complete graph has an even number of vertices, and hence F is a 1-factor, has been investigated, see for instance [**2, 3**]. In this talk I will survey the results on these conjectures and I will focus on some related open problems.

References

[1] P. Horak and A. Rosa. On a problem of Marco Buratti. *Electron. J. Combin.*, **16**: R20, 2009.

[2] D. Kohen and I. Sadofschi Costa. On a generalization of the seating couples problem. *Discrete Math.*, **339**: 3017–3019, 2016.

[3] M. Meszka, A. Pasotti and M.A. Pellegrini. The seating couple problem in even case. arXiv:2308.16553.

[4] A. Pasotti and M.A. Pellegrini. A generalization of the problem of Mariusz Meszka. *Graphs Combin.*, **32**: 333–350, 2016.

[5] E. Preissmann and M. Mischler. Seating couples around the King's table and a new characterization of prime numbers. *American Mathematical Monthly*, **116**: 268–272, 2009.

Hadamard matrices and spherical designs

Wednesday
9h00

Patrick Solé

Institut de Mathématiques de Marseille, Marseilles, France

(Joint work with Minjia Shi, Danni Lu, A. Armario, R. Egan and F. Ozbudak)

Abstract

Hadamard codes over finite rings are explored wrt their covering radius for the chinese euclidean distance. Generalized bent sequences, when they exist, provide a lower bound. Upper bounds are obtained by considering a spherical code attached to Hadamard matrices of Butson type. Under mild hypotheses, this code is spherical 2-design, which yields upper bound on its covering radius on the sphere. This bound, in turn, gives an upper bound on the covering radius of the Hadamard codes.

Some constructions of strongly regular graphs and digraphs

Tuesday 14h30

Andrea Švob

University of Rijeka - Faculty of Mathematics

(Joint work with Dean Crnković and Francesco Pavese)

Abstract

In this talk, we will present a connection between some finite incidence structures - partial geometric designs **[1]** or a $1\frac{1}{2}$–design **[3]**, special partially balanced incomplete block designs **[2]**, directed strongly regular graphs and strongly regular graphs. Further, we study their properties and give a technique for constructing directed strongly regular graphs by using strongly regular graphs that have a nice family of intriguing sets. The talk is based on the work presented in **[4]**.

References

[1] R. C Bose, S. S. Shrikhande and N. M. Singhi. Edge regular multigraphs and partial geometric designs. *Proc. Internat. Colloq. Combin. Theory* **17**:49–81, 1976.

[2] W. G. Bridges and M. S. Shrikhande. Special partially balanced incomplete block designs and associated graphs. *Discrete Math.* **9**:1–18, 1974.

[3] A. Neumaier. $t\frac{1}{2}$–designs. *J. Combin. Theory, Ser. A* **28**:226–248, 1980.

[4] D. Crnković, F. Pavese and A. Švob. Intriguing sets of strongly regular graphs and their related structures. *Contrib. Discrete Math.* **18**:66–89, 2023.

Subsquares of Latin squares

Monday
9h00

Ian Wanless

Monash University - School of Mathematics

(Including joint work with Jack Allsop)

Abstract

A *Latin square* is a matrix in which each row and column is a permutation of the same set of symbols. A *subsquare* is any submatrix which is itself a Latin square. Every Latin square of order n trivially has n^2 subsquares of order 1 and one subsquare of order n. Any subsquare between these two extremes is *proper*. Subsquares of order 2 are called *intercalates*. A Latin square without intercalates is said to be N_2 and a Latin square without proper subsquares is said to be N_∞.

In this talk I will survey results and open questions relating to the number of subsquares in a Latin square. We might be trying to minimise or maximise this number, or to understand its distribution among all Latin squares of a given order. The existence question for N_2 Latin squares was settled a long time ago, but the corresponding question for N_∞ Latin squares has only just been settled. There has also been exciting recent progress on understanding the distribution of intercalates among Latin squares of order n. But many questions remain.

INVITED POPULAR TALK

Mathematics and Flamenco: An Unexpected Partnership

Wednesday
18h00

JOSÉ-MIGUEL DÍAZ-BÁÑEZ

UNIVERSIDAD DE SEVILLA, SPAIN

Abstract

In this talk, we present a series of mathematical problems that throw interesting lights on flamenco music. More specifically, these are problems in discrete and computational mathematics suggested by an analytical examination of flamenco "cante" (singing). Flamenco music was born in Andalusia and is well known around the world. It has been declared an Intangible World Heritage by UNESCO. Flamenco music is a unique musical genre with an identity and originality worthy of scientific study. We pose several problems that analyzes flamenco rhythms and melodies and propose several possible answers based on a study that uses mathematics as a codifying operator. The talk is illustrated with live singing and dancing.

CONTRIBUTED TALKS

Grid-Based Graphs, Linear Realizations, and the Buratti-Horak-Rosa Conjecture

Tuesday
9h55

Onur Ağirseven

Unaffiliated (USA/Turkey)

(Joint work with M. A. Ollis, Emerson College, USA)

Abstract

Label the vertices of the complete graph K_v with the integers $\{0, 1, \ldots, v-1\}$ and define the *length* ℓ of the edge between distinct vertices labeled x and y by $\ell(x, y) = \min(|y-x|, v-|y-x|)$. A *realization* of a multiset L of size $v-1$ is a Hamiltonian path through K_v whose edge labels are L. The *Buratti-Horak-Rosa (BHR) Conjecture* is that there is a realization for a multiset L if and only if for any divisor d of v the number of multiples of d in L is at most $v-d$.

Using grid graphs, we construct particular types of realizations, called "linear realizations", for multisets with two distinct elements. Local modifications, often combined with path concatenations, then yield quick results, especially for multisets with up to five distinct elements. Our current focus is mainly on multisets whose underlying set has one of the following forms for small k: $\{1, x, x+k\}$, $\{1, k, x\}$ and $\{1, x, kx \pm 1\}$. These constructions considerably extend the parameters for which BHR Conjecture is known to hold.

On regular sets of affine type in finite Desarguesian planes and related codes

Tuesday
11h40

Angela Aguglia

Politecnico di Bari- Dept. of Meccanica, Matematica e Management, Italy

(Joint work with Bence Csajbók and Luca Giuzzi)

Abstract

In this talk we consider point sets of the finite Desarguesian plane $PG(2, q)$, with q any prime power, such that the multisets of intersection numbers obtained from different parallel classes of lines are the same for all but one parallel class of lines. We call such sets *regular of affine type*. When the lines of the exceptional parallel class have the same intersection numbers, then we call these sets *regular of pointed type*. Classical examples are e.g. unitals which are sets of $q^3 + 1$ points in $PG(2, q^2)$ which have the same intersection characteristics as Hermitian curves with respect to lines, i.e. they meet every line in either 1 or $q + 1$ points. A detailed study and constructions of such sets with few intersection numbers is due to Hirschfeld and Szőnyi from 1991, **[2]**. We here provide some general construction methods for regular sets and describe a few infinite families. The members of one of these families have the size as a unital and meet affine lines of $PG(2, q^2)$ in one of 4 possible intersection numbers, each of them congruent to 1 modulus $\sqrt{q}$. As a byproduct, we also determine the intersection sizes of the Hermitian curve defined over $GF(q^2)$, q a square, with suitable rational curves of degree $\sqrt{q}$ and we obtain $\sqrt{q}$-divisible codes with 5 non-zero weights. Finally, we exhibit the weight enumerator of the codes arising from the general constructions up to some q-powers and discuss some open problems, **[1]**.

References

[1] A. Aguglia, B. Csajbók, P. Sziklai and L. Giuzzi. On regular sets of affine type in finite Desarguesian planes and related codes. *Discrete Mathematics* **347**, 2024.

[2] J.W.P. Hirschfeld and T. Szőnyi. Sets in a finite plane with few intersection numbers and a distinguished point. *Discrete Mathematics* **97**, 1991.

Linear Systems of Conics over Finite Fields

Monday
12h30

Nour Alnajjarine

University of Rijeka - Faculty of Mathematics

(Joint work with Michel Lavrauw)

Abstract

Let $\mathcal{F}_2(2, \mathbb{F})$ be the space of 2-forms defined on $\mathrm{PG}(2, \mathbb{F})$. *Linear systems of conics* are subspaces of the projective geometry associated with $\mathcal{F}_2(2, \mathbb{F})$. In particular, 1, 2, 3 and 4-dimensional subspaces are respectively referred to as *pencils, nets, webs* and *squabs of conics*. The study of linear systems of conics enjoys a long and rich history, dating back to the mid-19th century. Previous results on classifying these systems have shown the insufficiency of relying solely on computational methods. It appears unlikely that this approach alone will resolve the long-standing open problem of classifying projectively inequivalent linear systems of conics over finite fields (see e.g. [**2, 4, 5**]. Therefore, to solve this problem, a more comprehensive examination of the various linear systems and their connections is necessary. This need for a new approach has triggered interest in broadening our understanding of linear systems of conics beyond merely classifying them. In this talk, we explain how the combinatorial and geometric properties of the quadric Veronesean in $\mathrm{PG}(5, q)$ can be utilized to determine a set of complete invariants of linear systems of conics in $\mathrm{PG}(2, q)$. In particular, we use the correspondence between webs of conics and lines of $\mathrm{PG}(5, q)$ to determine some characterizations of webs over finite fields, including the number of different types of conics contained in projectively inequivalent webs. Additionally, we give an interesting correspondence between 3×3 symmetric MRD-codes and certain types of webs of conics over finite fields of odd characteristic [**1**].

References

[1] N. Alnajjarine, M. Lavrauw. Webs and squabs of Conics over Finite Fields. Preprint.

[2] N. Alnajjarine, M. Lavrauw, T. Popiel. Solids in the space of the Veronese surface in even characteristic. *Finite Fields and Their Applications*, **83**: 102068, 2022.

[3] A. Campbell. Nets of conics in the Galois field of order 2^n. it Bull. Amer. Math. Soc, **34**: 481–489, 1928.

[4] M. Lavrauw and T. Popiel. The symmetric representation of lines in $\mathrm{PG}(\mathbb{F}_q^3 \otimes \mathbb{F}_q^3)$, *Discrete Math.* **343**: 111775, 2020.

[5] A. Wilson, The canonical Types of Nets of Modular Conics. *Amer. J. Math.*, **36**:187–210, 1914.

Cocyclic Two-Circulant Core Hadamard Matrices

Friday
11h15

Santiago Barrera Acevedo

La Trobe University - Department of Mathematical and Physical Sciences

(Joint work with Padraig Ó Cathaín and Heiko Dietrich)

Abstract

The construction of Hadamard matrices (HMs) using the two-circulant core (TCC) method relies on two sequences with almost perfect autocorrelation to generate a HM. A research problem posed by K. Horadam inquires whether such matrices are cocyclic. Exploring this question leads us to investigate the automorphism group of these combinatorial designs, which subsequently prompts a broader inquiry into the classification of transitive permutation groups of degree $2m+2$ containing an element of cycle type $1+1+m+m$, where $m \geqslant 1$ is an odd integer. In this presentation, I will outline this classification and show how it establishes conditions on the orders of cocyclic TCC HMs.

New advances on graph families associated with graphicable algebras.

Tuesday
11h15

Manuel Ceballos

Universidad Loyola Andalucía - Departamento de Ingeniería

Abstract

This paper focuses on the link between Graph Theory and graphicable algebras (a subclass of evolution algebras). Tian and Vojtechovsky introduced evolution algebras in 2006 in [**11**]. A couple of years later, the basic notions of evolution algebras were established by Tian [**10**]. Algebraically, they are Banach non-associative algebras, while in the realm of dynamics, they can be seen as discrete dynamical systems. There are many connections between evolution algebras and other mathematical fields such as genetics, group theory, stochastic processes, graph theory, physics, etc. ([**9**] can be seen, for example).

Graph Theory is an essential tool to deal with a wide range of problems in many mathematical areas. Regarding evolution algebras, Tian presented a method in 2008 [**10**] for establishing a connection between evolution algebras and graphs. This connection has been studied by many researchers such as Rozikov and Tian [**9**]; Elduque and Labra [**6**]; Cabrera, Siles and Velasco [**2**]; Cadavid, Rodiño and Rodríguez [**3, 4**]; Ceballos, Núñez and Tenorio [**1, 5**], etc.

Graphicable algebra, as a subset of evolution algebras, holds significant importance in mathematical research and applications. This algebraic structure provides a systematic framework for representing and analyzing evolutionary processes and relationships within dynamic systems. There exists also a very close relation between graphicable algebras and Graph Theory. Tian presented path, cycle and complete algebras as types of graphicable algebras, each corresponding to paths, cycles and complete graphs, respectively. Similarly, in [**8**], Núñez, Rodríguez, and Villar explored complete n-partite, friendship, star, snark, wheel and Petersen generalized graphicable algebras. They also introduced the concept of S-graphicable in [**7**].

The main goal of the present paper is to extend the research line initiated in [**8**] and [**7**] complementing the study developed in these papers. More concretely, we introduce new types of graphicable algebras associated to n-cubes, fullerene, platonic solid, Blanusa and Pappus graphs. Several relationships and subgraphicable algebras of some well-known families are also shown. We also analyze the type of graphicable algebra associated to the families of graphs considered in [**8**] and this paper. In particular, we explore the solvability of those algebras, which from a biological perspective, can be understood as indicating that some of the primary generators cease to exist after a specific number of generations. Finally, an algorithmic method has been implemented. This is devoted to checking if a given evolution algebra is S-graphicable and, in such a case, drawing its associated graph.

References

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Dimension of Affine Classes

Friday
10h50

Robert Craigen

University of Manitoba, Department of Mathematics

Abstract

Write U for the set of complex **units**, i.e., $U = \{e^{i\theta} \mid \theta \in \mathbb{R}\}$, the unit circle. A **Complex Hadamard matrix** (CHM) of order n is a matrix $M \in U^{n\times n}$ such that $MM^* = nI$, where M^* is the Hermitian adjoint of M The following are CHMs of orders $2, 3, 4$ respectively:

$$\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & \gamma & \gamma \\ \gamma & 1 & \gamma \\ \gamma & \gamma & 1 \end{pmatrix} \quad (\gamma = e^{\frac{\pi}{3}i}) \qquad \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & \lambda & -\lambda \\ 1 & -1 & -\lambda & \lambda \end{pmatrix} \quad (\text{any } \lambda \in U)$$

CHMs are preserved by permutation of rows and/or columns and multiplication of rows and/or columns by units (elements of U–equivalence relations for CHMs. Every CHM is equivalent to one which is **dephased** (first row and column consist of 1s.

Equivalence classes have long been of interest but CHMs also belong to **affine classes**–sets of CHM obtained by multiplying elements variables in U, whose **dimension** is essentially number of independent variables. For example, the third matrix is not a single matrix but a 1-dimensional affine class of dephased CHMs of order 4, found by Hadamard in 1893. (The other two shown have dimension 0 and are not dephased.) Affine CHM dimension is of interest in quantum information theory but makes an interesting study on its own terms. Tables of highest-known (dephased) affine dimension are available. We show how to improve previously known dimension for infinitely many orders. In particular we obtain dimension 13 in order 12 (the previous best in the online CHM catalogue [**1**] was 9).

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Constructing doubly even self-dual codes from Hadamard matrices

Wednesday
9h55

Dean Crnković

University of Rijeka - Faculty of Mathematics

(Joint work with Andrea Švob)

Abstract

In this talk, we will show that from every skew-type Hadamard matrix of order $4t$ one can obtain a series of skew-type Hadamard matrices of order $2^{i+2}t$, i a positive integer, whose binary linear codes are doubly even self-dual binary codes of length $2^{i+2}t$. It is known that a doubly even self-dual binary code yields an even unimodular lattice. Hence, this construction of skew-type Hadamard matrices gives us a series of even unimodular lattices of rank $2^{i+2}t$, i a positive integer. Furthermore, we provide a construction of doubly even self-dual binary codes from conference graphs.

New Spence Difference Sets

Thursday
11h15

James A. Davis

University of Richmond, VA, USA

(Joint work with John Polhill, Ken Smith, and Eric Swartz)

Abstract

There are five families of (v, k, λ)-difference sets with the property that $\gcd(v, k - \lambda) > 1$: Hadamard, McFarland, Spence, Davis-Jedwab, and Chen. The Hadamard and McFarland families have been studied extensively, the other three families less. The Spence family was discovered in 1977 **[2]**, and Drisko generalized this construction in 1998 **[1]** . Other than these constructions, we are not aware of any other constructions. We provide new examples in nonabelian groups, and we point to a new approach that will likely generalize to other families of difference sets.

References

[1] A. Drisko. Transversals in Row-Latin Rectangles. *J. Comb. Theory Ser. A* **84(2)**:181–195, 1998.

[2] E. Spence. A family of difference sets in non-cyclic groups. *J. Comb. Theory Ser. A* **22**:103–106, 1977.

Hadamard matrices in centraliser algebras of monomial representations

Friday
11h40

Ronan Egan

Dublin City University - School of Mathematical Sciences

(Joint work with Santiago Barrera Acevado, Heiko Dietrich and Padraig Ó Catháin)

Abstract

An $n \times n$ matrix H with complex entries of modulus 1 such that

$$HH^* = nI_n$$

is a (complex) Hadamard matrix of order n, where H^* denotes the complex conjugate transpose of H. When restricted to real entries, it is necessary that either $n = 1, 2$ or that $n = 4m$ for a positive integer m. It is long conjectured that this condition is sufficient but this is far from settled. Despite this, it appears that the number of (real) Hadamard matrices of order $4m$ grows exponentially, and classifications even at relatively small orders seem computationally infeasible. This motivates the restriction of our study to matrices with algebraic constructions, such as group development and cocyclic development.

Motivated by the work on monomial representations of Higman **[2, 3]**, in this talk, we will summarise and extend previous work on monomial group representations and their centraliser algebras. We will locate the theory of group developed and cocyclic Hadamard matrices within the study of such representations. Time permitting, we will apply techniques of computational algebra, e.g., Gröbner bases, to search for complex Hadamard matrices in the centraliser of a monomial representation. This is motivated by and extends the work of Chan **[1]** and Moorhouse **[4]** on permutation groups acting on complex Hadamard matrices.

References

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On the additive embedding of $\mathrm{PG}(2,q)$ into $\mathrm{AG}(3,q)$

Thursday 14h55

Giovanni Falcone

University of Palermo, Department of Mathematics and Computer science

Abstract

Recently [**1**] M. Buratti and A. Nakic provided a map, which embeds the projective space $\mathrm{PG}(n,q)$ into the affine space $\mathrm{AG}(n+1,q)$ with the property that the coordinate sum of the points of the images of (projective) hyperplanes is $O \equiv (0,\ldots,0) \in \mathrm{AG}(n+1,q)$. After a short comparison with the 3-dimensional real projective space, we show that this makes the point-hyperplanes 2-design of $\mathrm{PG}(n,q)$ a sub-design of the 1-$(q^{n+1}-1, \frac{q^n-1}{q-1}, r)$ design, where

$$r = \frac{1}{q^{n+1}}\left(\binom{q^{n+1}-1}{\frac{q^n-1}{q-1}} - (q^{n+1}-1)\binom{q^n-1}{\frac{q^{n-1}-1}{q-1}}\right),$$

whose points are all the non-zero $(n+1)$-tuples of $\mathrm{AG}(n+1,q)$, whose blocks are the $\frac{q^n-1}{q-1}$-subsets of points summing up to zero, and whose automorphisms are just the ones induced by the elements in $\mathrm{GL}(n+1,q)$ [**2**].

References

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[2] G. Falcone and M. Pavone. Permutations of zero-sumsets in a finite vector space. *Forum Math.*, **33 (2)**, publisher, 2021.

A linear programming bound for sum-rank-metric codes

Wednesday
11h40

Alexander L. Gavrilyuk

Shimane University - Department of Mathematics

(Joint work with Aida Abiad, Antonina P. Khramova and Ilia Ponomarenko)

Abstract

We will discuss how to compute a linear programming bound on the size of codes in the sum-rank metric. Our computational experiments show that this approach improves on all previously known bounds obtained by combinatorial or algebraic arguments (see, e.g., [**1**]).

References

[1] A. Abiad, A. Khramova, and A. Ravagnani. Eigenvalue bounds for sum-rank-metric codes. *IEEE Transactions on Information Theory.* In press, 2024.

On the combinatorial properties of shrinking sequences

Wednesday
14h55

Domingo Gómez-Pérez

Universidad de Cantabria

(Joint work with Ana I. Gómez and Veronica Requena)

Abstract

The shrinking families are different methods to construct pseudorandom bit generators based on the combination of linear feedback shift registers of maximum period. These are synchronized with a common clock and produce binary sequences with good statistical properties. Due to its simplicity and efficient implementation, the shrinking generator is particularly suitable for stream cipher cryptographic schemes. In this talk, we study the related linear codes generated by these sequences **[2]** and the underlying difference set that appears in the interleave structure **[1]**.

References

[1] D. Everett. Periodic digital sequences with pseudo-noise properties. *GEC J.*, **33 (3)**:115-118, 1966.

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Algebraically-informed deep networks for evolution algebras

Friday
12h30

Juan Núñez-Valdés

University of Seville - Department of Geometry and Topology

(Joint work with Desamparados Fernández-Ternero and Víctor M. Gómez-Sousa)

Abstract

In this paper, we extend the application of the algebraically-informed deep networks (AIDNs) introduced by Mustafa Hajij et al. in the case of evolution algebras. This deep learning algorithm allows us to represent any finite evolution algebra with a set of deep neural networks, through a formal set of algebraic relations that the algebra satisfy. The performance of AIDNs is tested for known theoretical results relative to associative evolution algebras. This is a first step towards neural-network-aided classification of evolution algebras in general, not only associative, which is the most common case.

References

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Critical sets based on non-trivial autoparatopisms of Latin squares

Monday 12h05

Manuel González-Regadera

Universidad de Sevilla

(Joint work with Raúl M. Falcón and María Dolores Frau)

Abstract

This paper deals with critical sets of Latin squares having a given paratopism in their autoparatopism group. It generalizes a similar problem concerning critical sets of Latin squares having a given isotopism in their autotopism group. This last problem has completely been solved for Latin squares of order up to five **[2]**, and also for order up to six when the mentioned autotopism is trivial **[1]**. Here, we introduce a new equivalence relation among partial Latin subsquares based on autoparatopism groups. It enables us to solve both problems for Latin squares of order up to six. Some particular cases of higher orders are also considered.

References

[1] P. Adams, R. Bean and A. Khodkar. A census of critical sets in the Latin squares of order at most six. *Ars Combinatoria* **68**: 203–223, 2003.

[2] R. M. Falcón, L. Johnson and S. Perkins. A census of critical sets based on non-trivial autotopisms of Latin squares of order up to five. *AIMS Mathematics* **6**: 261–295, 2021.

Exponentiation of Graphs

Toru Hasunuma

Tokushima University

Tuesday
10h50

Abstract

We newly introduce exponentiation of graphs. Let G and H be connected graphs of orders p and q, respectively. We then define the exponential graph G^H with base G and exponent H. From our definition, it follows that G^H has order $p^q q$, the minimum (maximum) degree of G^H is the sum of those of G and H, and G^H has logarithmic diameter if G has logarithmic diameter. We also show that G^H is maximally connected, i.e., the connectivity equals to the minimum degree.

As a doubly exponential scale network, DCell was proposed by Guo et al. **[2]** and its various properties have been investigated until now. The DCell network $D_{k,n}$ is a maximally connected $(n+k-1)$-regular graph of order at least $(n+\frac{1}{2})^{2^k}-\frac{1}{2}$ with diameter at most $2^{k+1}-1$. Exponentiation of graphs can be applied to construct very large scale networks with logarithmic diameter and highly fault-tolerance. De Bruijn networks are well-known to be bounded degree networks with logarithmic diameter **[1]**. For the complete graph K_n of order n and the d-ary de Bruijn (undirected) graph $B^*(d,k)$, the exponential graph $K_n^{B^*(d,k)}$ is a maximally connected graph of order $n^{d^k} \cdot d^k$ with maximum degree $n+2d-1$ and diameter $2d^k+k-2$. Compared to $D_{k,n}$, $K_n^{B^*(d,k)}$ has an advantage that the order can be increased while fixing the maximum degree. Moreover, we can construct multiexponential scale networks, e.g., $K_m^{K_n^{B^*(d,k)}} = K_m^{\left(K_n^{B^*(d,k)}\right)}$ is a maximally connected graph of order $m^{n^{d^k} \cdot d^k} \cdot n^{d^k} \cdot d^k$.

References

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Schur rings over $\mathrm{Sp}(\mathbf{n}, \mathbf{2})$ and multiplicity one subgroups

Thursday 15h50

Stephen Humphries

Brigham Young University, U.S.A. - Department of Mathematics

(Joint work with Grady Anderson, Andrea Barton and Nathan Nicholson)

Abstract

We study commutative Schur rings over the symplectic groups $\mathrm{Sp}(n, 2)$ containing the class $\mathcal{C}$ of symplectic transvections. We find the possible partitions of $\mathcal{C}$ determined by the Schur ring. We show how this restricts the possibilities for multiplicity one subgroups of $\mathrm{Sp}(n, 2)$. We also present work on determining multiplicity one subgroups of some special linear groups and symmetric groups.

References

[1] A. Barton and S. Humphries. Strong Gelfand pairs of $\mathrm{SL}(2, p)$. *Journal Alg. Appl.* **39**: 13 pages, 2023.

[2] G. Anderson, S. Humphries and N. Nicholson, Strong Gelfand pairs of symmetric groups. *Journal Alg. Appl.* **20**: 22 pages, 2021.

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New cyclic PBIBD(2)s obtained using finite field cyclotomic

Thursday 12h05

Laura Marie Johnson

University of St. Andrews - School of Mathematics and Statistics

(Joint work with S. Huczynska)

Abstract

PBIBD(2)s are partial analogues of BIBDSs and were first defined by Bose and Nair in [**1**]. The formal definition of a PBIBD(2) is given below.

Definition 1. Let G be an abelian group of cardinality v with a G-cyclic 2-class association scheme defined on it. A G-cyclic partially balanced incomplete block design with 2-associate classes (PBIBD(2)) is a;

(i) collection of b blocks,

(ii) each of cardinality k,

(iii) such that each element of v occurs in r blocks of the design,

(iv) and any two treatments that are i^{th} associates under the association scheme occur together in λ_i blocks of the design.

In [**2**], Wilson demonstrates that finite field cyclotomy can be used to obtain new PBIBD(n)s constructions. Building upon this idea, we will explore in this talk how cyclotomic constructions of objects known as Disjoint Partial Difference Families can be used to identify new constructions of PBIBD(2)s in finite fields of prime order.

References

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[2] R. M. Wilson. Cyclotomy and difference families in elementary abelian groups. *Journal of Number Theory*, **4**:17–47, 1972.

Strongly regular graphs decomposable into divisible design graph and Delsarte clique

Tuesday
15h25

Vladislav V. Kabanov

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Yekaterinburg, Russia

(Joint work with Alexander L. Gavrilyuk)

Abstract

In [**1**], Haemers and Higman studied a strongly regular graphs with a decomposition of their vertex sets in two parts such that the induced subgraphs on the parts are strongly regular, a clique, or a coclique. In [**2**], a prolific construction of strongly regular graphs was found: it is based on a regular decomposition of a divisible design graph (see [**3**]) with certain parameters and a clique, or a coclique. In [**4**], we determined the parameters of strongly regular graphs which admit a decomposition into a divisible design graph with any parameters and a coclique. In particular, it was shown that when the least eigenvalue of such a strongly regular graph is a prime power, its parameters coincide with those of the complement of a symplectic graph. As a counterpart of this result, we determine the parameters of all strongly regular graphs that can be decomposed into a divisible design graph and a Delsarte clique. In particular, an infinite family of strongly regular graphs with the required decomposition and a new infinite family of divisible design graphs are found.

References

[1] W. H. Haemers, D. G. Higman. Strongly Regular Graphs with Strongly Regular Decomposition. *Linear Algebra Appl.*, **114–115**:379–398, 1989.

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Loops with squares in two nuclei

Monday
9h55

Michael Kinyon

University of Denver - Department of Mathematics

(Joint work with J.D. Phillips (Northern Michigan University))

Abstract

The varieties of loops (quasigroups with identity elements) of Bol-Moufang type are defined by identities with the following properties: (i) they involve three distinct variables, each occurring on both sides of the equal sign; (ii) the variables occur in the same order on both sides; (iii) exactly one of the variables appears twice on both sides. For example, the identity $(xy)(zx) = x((yz)x)$ is of Bol-Moufang type and defines the variety of Moufang loops. (This is half of the rationale for the name "Bol-Moufang type").

Various interesting varieties of loops (interesting, at least, to quasigroup theorists) – such as Moufang loops, Bol loops, C loops and others – can be defined by identities of Bol-Moufang type. There are others which, up until now, have not been garnered much interest because there does not seem to be much that can be said about them. For instance, the variety of loops with *left nuclear squares* is defined by $(xx)(yz) = ((xx)y)z$; essentially nothing interesting can be proven about these or the similarly defined varieties of loops with middle nuclear or right nuclear squares.

It turns out however, that one can say interesting things about the pairwise intersection of these varieties (from whence comes the title of this talk). This is a bit surprising because on the face of it, the condition that squares associate with all other elements in certain positions does not seem like a very strong property.

In particular, it turns out that in such loops, the intersection of the corresponding nuclei is a normal subloop. In addition, we consider the subvariety of loops with *central* squares. In such loops, having the automorphic inverse property (AIP) $(xy)\backslash 1 = (x\backslash 1)(y\backslash 1)$ is equivalent to endomorphic squaring $(xy)^2 = x^2y^2$. In the finite case, such loops decompose into a direct product into an abelian group of odd order and a loop consisting of elements of order a power of 2. This is analogous to what happens in other Bol-Moufang type varieties in the presence of the AIP.

On Latin Young Diagrams

Monday
11h15

Daniel Kotlar

Tel-Hai College - Department of Computer Science

(Joint work with Ron Aharoni, Eli Berger and He Guo)

Abstract

A Young diagram is called *Latin* if there is an assignment of integers to its cells so that in each row i of length a_i the numbers $1, \ldots, a_i$ appear, and the assignment is injective in each column. A Young diagram Y is *wide* if every sub-diagram Z formed by a subset of the rows of Y dominates Z', the conjugate of Z. Wideness is necessary for being Latin. The next conjecture is attributed in **[1]** to Chow and Taylor.

Conjecture 1 (Wide Partition Conjecture, or WPC)**.** *If a Young diagram is wide then it is Latin.*

We assign to a Young diagram Y a tripartite 3-hypergraph $H(Y)$, whose three sides are the rows $\{r_1, \ldots, r_m\}$, the columns $\{c_1, \ldots, c_{a_1}\}$, and the symbols $\{1, \ldots, a_1\}$, and whose edges are $\{\{r_i, c_j, s\} \mid 1 \leq i \leq m, 1 \leq j, s \leq a_i\}$.

Following notation from **[2]**, a 2-matching in a hypergraph H is a set of edges, every two of which share fewer than 2 vertices. We denote by $\nu^{(2)}(H)$ the maximal size of a 2-matching in H. Note that for a Young diagram Y, having a Latin assignment is equivalent to $\nu^{(2)}(H(Y)) = |Y|$, where $|Y|$ is the total number of cells in Y. Thus, WPC is equivalent to

Conjecture 2. *If a Young diagram Y is wide then $\nu^{(2)}(H(Y)) = |Y|$.*

A 2-cover in a hypergraph H is a set P of pairs of vertices of H that covers all edges of H, that is, for each edge e in H there exists a pair in P that is contained in e. We denote by $\tau^{(2)}(H)$ the minimal size of a 2-cover of H. It is easy to note that $\tau^{(2)}(H) \geq \nu^{(2)}(H)$, and for a Young diagram Y, $\tau^{(2)}(H(Y)) \leq |Y|$. Our main result is the following weaker version of WPC:

Theorem 3. *If a Young diagram Y is wide then $\tau^{(2)}(H(Y)) = |Y|$.*

References

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On mosaics of designs

Thursday
10h50

Vedran Krčadinac

University of Zagreb, Croatia

Abstract

Let t_i-(v, k_i, λ_i), $i = 1, \ldots, c$ be parameters of combinatorial designs, all with v points and b blocks. A *mosaic* t_1-$(v, k_1, \lambda_1) \oplus \cdots \oplus t_c$-$(v, k_c, \lambda_c)$ is a $v \times b$ matrix with entries from $\{1, \ldots, c\}$ such that the entries i represent incidences of a t_i-(v, k_i, λ_i) design. Mosaics of designs were introduced in **[2]** and a construction from resolvable designs was presented. Mosaics of symmetric designs can be obtained from tilings of groups with difference sets **[1]**. These constructions give rise to *homogenous* mosaics, meaning that the component designs have equal parameters. We shall present several new constructions for mosaics that are not homogenous, homogenous mosaics comprising designs that are not resolvable, and mosaics of symmetric designs not coming from difference sets. Mosaics of designs have applications for the generation of information-theoretic security in communication and data storage systems **[3, 4]**.

References

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Skew-adjacency matrices of tournaments with bounded principal minors

Thursday 17h10

SOUKAÏNA MAHZOUM

HASSAN II UNIVERSITY OF CASABLANCA - FUNDAMENTAL AND APPLIED MATHEMATICS DEPARTMENT / FACULTY OF SCIENCES AIN CHOCK / CASABLANCA, MOROCCO

(Joint work with Abderrahim Boussaïri, Sara Ezzahir and Soufiane Lakhlifi)

Abstract

A tournament is a digraph in which every pair of vertices is jointed by exactly one arc. Let T be a tournament with n vertices $v_1, \ldots, v_n$. The skew-adjacency matrix of T is the $n \times n$ zero-diagonal matrix $S = [s_{ij}]$ in which $s_{ij} = -s_{ji} = 1$ if v_i dominates v_j. It is well-known that the determinant of S is zero or the square of an odd integer. Moreover, the principal minors of S are at most 1 if and only if T is a local order. In this paper, we characterize the class of tournaments for which the principal minors of the skew-adjacency matrix do not exceed 9.

References

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On the Hadamard multiary quasigroup product

Monday
11h40

LORENZO MELLA

UNIVERSITY OF MODENA AND REGGIO EMILIA

(Joint work with R.M. Falcón and P. Vojtěchovský)

Abstract

The Hadamard product between Latin squares has been introduced in **[1]** as a generalization of the Hadamard product between two matrices. For an integer $n \geq 1$, given two $n \times n$ arrays A, B and a Latin square L of order n filled with elements from $\{1, \ldots, n\}$, the Hadamard product between A and B through L, written as $A \odot_L B$, is defined as the array having $L[A[i,j], B[i,j]]$ in its (i,j)-th entry. Under suitable conditions on A, B and L, proved in **[1]**, the array $A \odot_L B$ is a Latin square. In this talk, based on the joint work **[2]**, we show that these conditions highlight a connection with orthogonal Latin squares, and we discuss the generalization of this operator to multiary quasigroups.

References

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Resolvable $\lambda-$Golomb rules and resolvable cyclic configurations.

Wednesday 15h50

CRISTIAN C. MENESES G.

UNIVERSIDAD DEL CAUCA - DEPARTMENT OF MATHEMATICS/ SCHOOL OF EDUCATION

(Joint work with Carlos A. Martos O. and Daza, D. F.)

Abstract

A Golomb ruler is a set of non-negative integers $a_1 < a_2 < \cdots < a_k$ such that all differences $a_j - a_i$ (with $j \neq i$) are distinct. This concept has been studied in the cyclic group of residues modulo M, denoted as $\mathbb{Z}_M$. In this group, Golomb rulers are referred to as modular Golomb rulers. In 2020, Buratti and Stinson introduced the concept of a *resolvable Golomb ruler* [**1**], a special category of Golomb rulers that satisfies the "resolvability" condition, where the elements of the ruler cover all residue classes modulo the order of the ruler.

Past research [**2,3,4,6**] found a relationship between Golomb rulers and various combinatorial structures such as the cyclic λ-configurations and resolvable cyclic configurations. A cyclic λ-configuration with parameters $(v_k)_\lambda$ consists of a set of v points and a set of v lines, where each line contains exactly k points and each point is present in exactly k lines. The connection between two points is limited to a maximum of λ lines, and the intersection between two lines is restricted to a maximum of λ points. Moreover, each line is a translation of the points of the first line. A λ-configuration with parameters $(v_k)_\lambda$ is resolvable if the set of blocks can be partitioned into r parallel classes, each consisting of v/k blocks that partition the set of points. Since the main problem in the study of λ-configurations is finding existence parameters (v, k, λ), Buratti and Stinson [**1**] showed that a (resolvable) modular Golomb ruler is equivalent to a (resolvable) cyclic 1-configuration.

Following the generalization of the Golomb ruler concept - where at most λ repetitions are allowed in the differences of its elements, known as a λ-Golomb ruler - this work generalizes the concept of a resolvable Golomb ruler. Additionally, following the method of Buratti and Stinson [**1**], it is shown that a λ-Golomb ruler is equivalent to a cyclic λ-configuration, and that a resolvable λ-Golomb ruler is equivalent to a resolvable λ-cyclic configuration. Furthermore, the constructions of modular λ-Golomb rulers presented by Ojeda, Urbano, and Solarte in [**5**] help to obtain new parameters for λ-cyclic configurations and resolvable cyclic λ-configurations.

References

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Additive Graph Decompositions

Thursday
9h55

Francesca Merola

Roma Tre University (Italy) — Department of Mathematics and Physics

(Joint work with Marco Buratti and Anamari Nakić)

Abstract

A $t-(v,k,\lambda)$ design is additive if, up to isomorphism, the point set is a subset of an abelian group G and every block is zero-sum in G. This definition was introduced in **[2]** and was the starting point of an interesting new theory, see also for instance [**1, 3**].

One might generalize this concept in a natural way to graph decompositions as follows: an *additive graph decomposition* is a decomposition of a graph K into subgraphs $\Gamma_1, \Gamma_2, \ldots, \Gamma_t$ such that the vertex set $V(K)$ is a subset of an abelian group G and the sets $V(\Gamma_1), V(\Gamma_2), \ldots V(\Gamma_t)$ are zero-sum in G.

I will present some results and techniques for constructing additive graph decomposition, in particular in the cases of cycle and path decompositions of the complete graph.

References

[1] M. Buratti, A. Nakić, Super-regular Steiner 2-designs. *Finite Fields Appl.* **85**: Article number 102116 29pp, 2023.

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Regular digraphs and related linear codes

Wednesday
12h05

Vedrana Mikulić Crnković

University of Rijeka - Faculty of mathematics

(Joint work with Ivona Traunkar and Matea Zubović Žutolija)

Abstract

In this talk we present a method for constructing regular digraphs from a transitive permutation group, which is a generalisation of a construction method described in [**1**]. We use this method to construct directed quasi-strongly regular graphs from transitive permutation groups of degree up to 30.

Furthermore, we construct self-orthogonal and LCD codes over an arbitrary field by extending the adjacency matrices and using construction methods similar to those in [**2**].

References

[1] met D. Crnković, V. Mikulić Crnković and A. Švob. On some transitive combinatorial structures constructed from the unitary group $U(3,3)$. *J. Statist. Plann. Inference*, **144**:19-40, 2014.

[2] V. Mikulić Crnković and I. Traunkar. Self-orthogonal codes constructed from weakly self-orthogonal designs invariant under an action of M_{11}. *Applicable algebra in engineering communication and computing*, **34(1)**:139-156, 2023.

Some results on Graphic Topology defined on Tournaments

Friday
12h05

Inés Mora-Caro

University of Seville - Department of Geometry and Topology

(Joint work with Desamparados Fernández-Ternero)

Abstract

A large number of researchers have studied the problem of topologization of combinatorial structures, for example in [**3, 5**]. Within the framework of topologies defined on locally finite graphs, the graphic topology was developed in [**1, 2, 4**]. We continue the research about the graphic topology defined on finite indecomposable tournaments (complete digraphs), begun in [**1**].

We deduce a characterization of indecomposable tournaments with few vertices. We verify that the minimum number of vertices such that there exist non-isomorphic indecomposable tournaments with homeomorphic graphic topologies is six, viewing all graphic topologies on indecomposable tournaments from three to six vertices. We also prove that the family of non-homeomorphic graphic topologies on six vertices irreducible tournaments is strictly included in the family of non-isomorphic T_0 topological spaces with six points.

This work is a starting point for the study of graphic topology on tournaments with any number of vertices.

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Hadamard Partitioned Difference Families

Thursday
11h40

Anamari Nakic

University of Zagreb

Abstract

A $(G, [k_1, \ldots, k_t], \lambda)$ *partitioned difference family* (PDF) is a partition $\mathcal{B}$ of an additive group G into sets (*blocks*) of sizes $k_1, \ldots, k_t$, such that the list of differences of $\mathcal{B}$ covers exactly λ times every non-zero element of G. It is called *Hadamard* (HPDF) if the order of G is 2λ. The study of HPDFs is motivated by the fact that each of them gives rise, recursively, to infinitely many other PDFs. Apart from the *elementary* HPDFs consisting of a Hadamard difference set and its complement, only one HPDF was known. In this work we present three new examples in several groups and a start of a general investigation on the possible existence of HPDFs with assigned parameters by means of simple arguments.

Linearly equivalent flag codes

Wednesday
11h15

Miguel Ángel Navarro-Pérez

University Carlos III of Madrid - Department of Mathematics

(Joint work with Xaro Soler-Escrivá)

Abstract

The use of *flag codes* in the *Network Coding* setting was introduced in **[4]**. These codes were presented as a generalization of *constant dimension codes*. In this new context, codewords are *flags*, i.e., nested sequences of vector subspaces of a given finite dimensional vector space over a finite field.

Given a flag code, its *projected codes* were defined in **[3]** as the constant dimension codes containing all the subspaces of prescribed dimensions that form the flags in the flag code. During the last years, several papers have been focused on the study and construction of flag codes, and also on the relationship between their parameters (minimum distance or bounds for the cardinality) and the ones of their projected codes **[1, 2, 3, 6]**.

In this talk, based on our work **[5]**, we introduce the notion of *linear equivalence* for flag codes and we compare it to the idea of linear equivalence for constant dimension codes **[7]**. First, we prove that linearly equivalent flag codes have linearly equivalent projected codes, but the converse does not always hold. Inspired by this question, we pay attention to the family of flag codes for which equivalence for their projected codes implies the flag codes equivalence. Derived from our study, we provide some new results concerning the automorphism group of certain families of flag codes, including optimum distance flag codes.

References

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Complete Solutions to the Uniform Hamilton-Waterloo Problem

Tuesday
12h05

Sibel Özkan

Gebze Technical University - Department of Mathematics

(Joint work with Mariusz Meszka)

Abstract

The Hamilton-Waterloo problem with uniform cycle sizes asks for a 2-factorization of the complete graph K_v (for odd v) or K_v minus a 1-factor (for even v), where r of the factors consist of m-cycles and s of the factors consist of n-cycles for some integers m and n satisfying necessary conditions with $r + s = \left\lfloor \frac{v-1}{2} \right\rfloor$.

Uniform version of the Hamilton-Waterloo Problem is a well studied problem and the general solutions for even cycle lengths, odd cycle lengths and different parity cycle lengths are mostly given (see **[1–4]**). In this work, we study the exceptions on the general solutions and cover them for certain cycle sizes and completing the solutions.

References

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On the maximum field of linearity of linear sets

Thursday
15h25

Valentina Pepe

Sapienza University of Rome

(Joint work with B.Csajbók and G. Marino)

Abstract

Let V denote an r-dimensional $\mathbb{F}_{q^n}$-vector space. A point set L of $\mathrm{PG}(V, q^n) = \mathrm{PG}(r-1, q^n)$ is called an $\mathbb{F}_q$-*linear set* of rank m if it is defined by the non-zero vectors of a m-dimensional $\mathbb{F}_q$-vector subspace U of V, i.e.

$$L = L_U = \{\langle \mathbf{u} \rangle_{\mathbb{F}_{q^n}} : \mathbf{u} \in U \setminus \{\mathbf{0}\}\}.$$

For a point $P = \langle \mathbf{u} \rangle_{\mathbb{F}_{q^n}} \in \mathrm{PG}(V, q^n)$ the *weight* of P with respect to the linear set L_U is

$$w_{L_U}(P) := \dim_q \langle \mathbf{u} \rangle_{\mathbb{F}_{q^n}} \cap U.$$

Let L_U be a linear set such that $w_{L_U}(P) > 1$ for every $P \in L_U$, then prove the following:

Theorem 4. *There exists* $1 < d|n$ *such that* $L_U = L_{U'}$*, where* $U' = \langle U \rangle_{\mathbb{F}_{q^d}}$*, that is* $L = L_U$ *is an* $\mathbb{F}_{q^d}$*-linear set for some* $d > 1$.

We will show some applications on coding theory, on the problem of the directions determined by an affine set and on Desarguesian spreads of projective spaces.

References

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On designs of degree 3

Thursday
12h30

Lucija Relić

University of Zagreb, Croatia

(Joint work with Vedran Krčadinac)

Abstract

A design is *schematic* if the set of its blocks with relations defined by intersection sizes is an association scheme. We consider 3-designs of degree 3, i.e. with three distinct intersection sizes. These designs do not satisfy the condition of the Cameron-Delsarte theorem and need not to be schematic. However, several known families of such designs are schematic. One of them arises from linked systems of symmetric designs, with the number of points being a power of 2 with even exponent **[1, 2]**. We will present a table of small admissible parameters and identify a similar family with the number of points a power of 2 with odd exponent. Designs in this new family are not schematic.

References

[1] P.J. Cameron and J.J. Seidel. Quadratic forms over GF(2). *Indag. Math.*, **76**:1–8, 1973.

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Determining exact values of 4-color-off-diagonal generalized Schur numbers

Thursday
16h45

M. Pastora Revuelta

University of Seville - Department of Applied Mathematic I

(Joint work with T. Ahmed, L. Boza and M.I. Sanz)

Abstract

Given positive integers r and k_i, with $r \geq 2$ and each $k_i \geq 2$ for $i = 1, \ldots, r$, the concept of the r-color off-diagonal generalized Schur number, denoted by $S(r; k_1, k_2, \ldots, k_r)$, emerges as a critical measure in combinatorial number theory. This number is defined as the smallest positive integer M such that any r-coloring of the set of integers $[1, M]$ inevitably contains a monochromatic solution to the equation $E_{k_j} : x_1 + x_2 + \cdots + x_{k_j} = x_{k_{j+1}}$ for at least one color j within the range 1 to r.

The study of such numbers began with notable contributions such as those by Baumert **[2]** in 1961, who established the value of $S(4; 2, 2, 2, 2)$, and continued with Ahmed and Schaal **[1]** in 2016, who identified values for $S(4; 2, 2, 2, 3)$ and $S(4; 2, 2, 2, 4)$.

This work presents an extensive expansion of known values, introducing 65 new exact values for $S(4; 2, k_1, k_2, k_3)$. These include:
$S(4; 2, 2, 2, k_3)$, where $5 \leq k_3 \leq 17$; $S(4; 2, 2, 3, k_3)$, where $3 \leq k_3 \leq 14$; $S(4; 2, 2, 4, k_3)$, where $4 \leq k_3 \leq 11$; $S(4; 2, 2, 5, k_3)$, where $5 \leq k_3 \leq 9$; $S(4; 2, 2, 6, k_3)$, where $6 \leq k_3 \leq 7$; $S(4; 2, 3, 3, k_3)$, where $3 \leq k_3 \leq 11$; $S(4; 2, 3, 4, k_3)$, where $4 \leq k_3 \leq 8$; $S(4; 2, 3, 5, k_3)$, where $5 \leq k_3 \leq 7$; $S(4; 2, 3, 6, 6)$; $S(4; 2, 4, 4, k_3)$, where $4 \leq k_3 \leq 7$;
$S(4; 2, 4, 5, k_3)$, where $5 \leq k_3 \leq 6$; and $S(4; 2, 5, 5, 5)$.

We define $F_2 = E_2$, and for each $k \geq 3$, $F_k : px_1 + qx_2 + (k - p - q)x_3 = x_4$, where $0 \leq p \leq q \leq (k - p)/2$.

We denote $S_*(r; k_1, k_2, \ldots, k_r)$ as the smallest integer m such that any r-coloring of $[1, m]$ provides a solution in one color to an equation from F_{k_i}. Given that F_k equations are specific cases of those in E_k, it follows that $S(r; k_1, k_2, \ldots, k_r)$ is always less than or equal to $S_*(r; k_1, k_2, \ldots, k_r)$.

In this work we propose the following conjecture:

Conjecture: *If* $2 \leq k_1 \leq k_2 \leq k_3$, then $S(4; 2, k_1, k_2, k_3)$ equals $S_*(4; 2, k_1, k_2, k_3)$.

This conjecture is supported by the previously known values and further validated by the new findings in this study.

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Direct product group codes and derived quantum codes

Wednesday
14h30

Miguel Sales Cabrera[1]

Universitat d'Alacant - Departament de Matemàtiques

(Joint work with Xaro Soler-Escrivà and Víctor Sotomayor)

Abstract

Let $\mathbb{F}_q$ be a finite field of q elements, and G a group of order n. A *group code* or *G-code* is a linear code that can be realised as a left ideal in the group algebra $\mathbb{F}_q[G]$ by mapping every canonical vector of the basis of $\mathbb{F}_q^n$ to a different element of G. This family of codes contains countless good codes [**1, 2, 6**], and many methods from representation theory can be applied. For instance, if the Wedderburn-Artin decomposition of $\mathbb{F}_q[G]$ is known, then every G-code can be enumerated, and its dimension can be computed easily.

The aim of this talk is twofold: firstly, we will present group codes when G is a direct product of a cyclic and a dihedral group, or two dihedral groups. It will be shown that some of these codes achieve the best known distance for their dimension. Secondly, quantum CSS codes [**3, 4**] will be constructed from these types of codes. The CSS construction of quantum codes is amongst the most popular techniques of obtaining quantum codes from classical codes. Researchers have been able to derive several efficient quantum codes, which are indispensable for the proper operation of quantum computers [**5**].

References

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Hemisystems and Strongly Regular Graphs

Tuesday 15h50

Valentino Smaldore

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(Joint work with V. Pallozzi Lavorante and F. Romaniello)

Abstract

In [**3, 4, 6**], there were constructed two non-isomorphic families of hemisystems of $H(3,p^2)$: the Cossidente-Penttila hemisystem [**3**], for every p prime power, which is stabilized by $PSL(2,p^2)$; and the Korchmáros-Nagy-Speziali hemisystem, constructed firstly in [**4**], and extended in [**6**], for every prime p of the form $p = 1+4a^2$, which is stabilized by $PSL(2,p) \times C_{\frac{p+1}{2}}$. In the case $p = 5$, the latter has full automorphism group $3.A_7$, and it is isomorphic to a sporadic hemisystem described in [**3**]. Here, we investigate the new family of hemisystems and the related strongly regular graphs. In this way, we find a new family of srg, cospectral but not isomorphic to the Cossidente-Penttila $srg(\frac{(q^3+1)(q+1)}{2}, \frac{(q^2+1)(q-1)}{2}, \frac{q-3}{2}, \frac{(q-1)^2}{2})$. Moreover we find connection between hemisystems, strongly regular graphs and two-weight linear codes.

References

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Alphabet reduction pairs of tables

Thursday 14h30

Sophie Toulouse

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(Joint work with Jean-François Culus)

Abstract

Table 1: A $(3,2)$-ARPA (P,Q) and a $(3,2)$-CPA (D,N) of strength 2.

P^0	P^1	P^2	Q^0	Q^1	Q^2	D^0	D^1	D^2	N^0	N^1	N^2
0	1	0	0	1	2	1	1	0	1	1	1
0	0	2	0	0	0	1	0	1	1	0	0
1	1	2	1	1	0	0	1	1	0	1	0
1	0	0	1	0	2	0	0	0	0	0	1

For $k > 0$, $p \geq k$, $q > p$, two arrays P and Q with q columns over symbol set $\Sigma_q := \{0, 1, \ldots, q-1\}$ form a (q,p)-*alphabet reduction pair of arrays — a* (q,p)-*ARPA for short — of strength* k if they coincide on any k-cardinality subset of their columns, and if the rows of P each involve at most p distinct symbols, while $(q) := 0\ 1\ \ldots\ q-1$ occurs at least once as a row in Q.

ARPAs allow to reduce optimization *Constrained Satisfaction Problems with bounded constraint arity* — a.k.a. k CSPs — over Σ_q to k CSPs over a smaller alphabet Σ_p [**1**]. In this context, we seek ARPAs maximizing the frequency of row (q) in Q. For example, in the ARPA shown in Table 1, this frequency is $1/4$. We show that with respect to this specific optimization criterion, ARPAs are equivalent to a seemingly simpler family of combinatorial designs that we introduce as *Cover pairs of arrays* (CPA for short). Two arrays D and N with q columns and coefficients in Σ_2 form a (q,p)-*CPA of strength* k if they coincide on any subset of cardinality k of their columns, and if the rows of D each involve at most p non-zero coefficients, while $1\ 1\ \ldots\ 1$ appears at least once as a row in N (see Table 1 for an example). We provide optimal ARPAs when either $p = k$ or $k \in \{1, 2\}$. Additionally, we expose constructions and bounds for ARPAs, as well as for their relaxed version in which two words $u_0\ u_1\ \ldots\ u_{q-1}$ and $u_0 + a\ u_1 + a\ \ldots\ u_{q-1} + a$ are considered equivalent.

We emphasis that each of the three considered families of combinatorial designs has direct consequences on the approximability of k CSPs.

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Contextual configurations

Stefan Trandafir

Simon Fraser University

(Joint work with Adán Cabello and Petr Lisoněk)

Wednesday
15h25

Abstract

Contextuality, a phenomenon predicted by quantum physics, has recently been identified as a key resource for quantum computing. One particular structure which captures this phenomenon is given by an evendegree hypergraph labeled with operators from the n-qubit Pauli group satisfying certain requirements. The two most well-known such structures are the 2-qubit Mermin Square, a $(9_2, 6_3)$ configuration, and the 3-qubit Peres-Mermin Pentagram, a $(10_2, 5_4)$ configuration. A result of Arkhipov (2012) shows that if a contextual hypergraph is 2-regular, then it admits a labeling with 2-qubit or 3-qubit Pauli operators by reducing to the Square or Pentagram. Prior to our work, all contextual hypergraphs known could be reduced to one of these cases. We present an efficient algorithm that checks whether a hypergraph admits a contextual labeling, and if so it generates a labeling with the minimum number of qubits. By considering contextual hypergraphs with vertices of degree greater than two, we found examples that cannot be reduced to the Square or Pentagram. Of these, there is the (21_4) Grünbaum-Rigby configuration (which requires 4 qubits), a 3-astral 4-configuration (which requires 4 qubits), the smallest known weakly flag-transitive configuration (which requires 5 qubits), as well as a (27_4) configuration (which requires 3 qubits). Our hope is to find an infinite family of contextual configurations that cannot be reduced.

LCD codes related to some combinatorial structures

Wednesday
12h30

Ivona Traunkar

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(Joint work with Vedrana Mikulić Crnković)

Abstract

A linear code $\mathcal{C}$ is called LCD code if $\mathcal{C} \cap \mathcal{C}^{\perp} = \{0\}$. Matrix G generates an LCD code if and only if $\det(G \cdot G^T) \neq 0$ (see **[1]**). A 1-design is weakly p-self-orthogonal if all the block intersection numbers gives the same residue modulo p.

In **[2]**, we analyse extensions of the incidence matrix, orbit matrix and submatrices of orbit matrix of a weakly p-self-orthogonal 1-design in order to construct self-orthogonal codes. We extend the methods of construction described in order to construct LCD codes.

We will present examples of LCD codes constructed from some weakly p-self-orthogonal designs.

References

[1] J. L. Massey. Linear codes with complementary duals. *Discrete Math.* **106/107**: 337–342, 1992.

[2] V. Mikulić Crnković and I. Traunkar. Self-orthogonal codes constructed from weakly self-orthogonal designs invariant under an action of M_{11}. *Applicable algebra in engineering communication and computing*, **34(1)**: 139–156, 2023.

Graphs of Latin Regular Hexahedra

Monday 10h50

Akihiro Yamamura

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(Joint work with Haruki Fukaura)

Abstract

Another generalization of a Latin square [**1**], called a *Latin regular hexahedron*, is introduced and shown its existence in [**3**]. A concrete construction using related combinatorial structures called a *Latin three-axis design* is given in the paper. It is shown that the construction does not produce all of the Latin regular hexahedra. We examine the construction for Latin regular hexahedra of small order. We call a Latin regular hexahedron defined using Latin three-axis designs *separable* and *inseparable* otherwise. We show that all Latin regular hexahedra of order 2 are separable using graph theoretical method, whereas there exists an inseparable Latin regular hexahedra of order 4. We are also interested in how many separable and inseparable Latin regular hexahedra of small order. Using graphs of Latin regular hexahedra and applying Cauchy-Frobenius lemma, we count the number of Latin regular hexahedra of order 2. Then we verify our counting is correct by computer experiment. The investigation concludes that every Latin regular hexahedron of order 2 is separable.

The existence of a Latin three-axis design and a Latin four-axis design are equivalent to 1-factorizations [**2**] of the complete tripartite graph $K_{2n,2n,2n}$ and the complete quadripartite graph $K_{n,n,n,n}$, respectively. In addition, we discuss perfect 1-factorization of some of these graphs. In particular, we show that every 1-factorization of $K_{2,2,2}$ is perfect.

References

[1] J.Dénes and A.D.Keedwell. *Latin Squares and their Applications New Developments in the Theory and Applications*, 2nd Edition, Elsevier, 2015.

[2] W.Wallis. *One-Factorizations.* Kluwer Academic Publishers, 1997.

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On Some Cases of the Directed Uniform Hamilton-Waterloo Problem

Tuesday 12h30

Fatih Yetgin

Gebze Technical University - Department of Mathematics

(Joint work with Uğur Odabaşı and Sibel Özkan)

Abstract

The directed Hamilton-Waterloo problem involves the decomposition of the complete symmetric digraph K_v^* into two non-isomorphic directed cycle factors. In its uniform version, these factors consist of directed m-cycles or n-cycles. If r and s represent the number of factors of directed m-cycles and n-cycles, respectively, then $r + s = v - 1$.

First, basic 2-factorization problems, such as the Oberwolfach and Hamilton-Waterloo problems **[1, 2]**, and their directed counterparts will be illustrated with examples. Then, we will explore solutions to the Directed Uniform Hamilton-Waterloo problem with two cycle lengths and introduce the constructions for doing so **[3, 4]**.

References

[1] B. Alspach, P.J. Schellenberg, D.R. Stinson and D. Wagner. The Oberwolfach problem and factors of uniform odd length cycles. *J. Comb. Theory Ser. A* , **52**:20–43, 1989.

[2] P. Adams, E. J. Billington, D. E. Bryant and S. I. El-Zanati. On the Hamilton-Waterloo problem. *Graphs and Combinatorics*, **18**:31-51, 2002.

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[4] F. Yetgin, U. Odabaşı and S. Özkan. The Directed Uniform Hamilton-Waterloo Problem Involving Even Cycle Sizes. *Discussiones Mathematicae Graph Theory*, **Accepted**:2024.

Storage Codes on Triangle-Free Graphs

Wednesday
10h50

Qing Xiang

Southern University of Science and Technology

(Joint work with Haihua Deng, Hexiang Huang and Guobiao Weng)

Abstract

Consider a simple, connected graph Γ with n vertices. Let C be a code of length n with its coordinates corresponding to the vertices of Γ. We define C as a *storage code* on Γ if, for any codeword $c \in C$, the information at each coordinate of c can be recovered by accessing its neighboring coordinates. The main problem here is to construct high-rate storage codes on triangle-free graphs. In this paper, we employ the polynomial method to address a question asked by Barg and Zémor in 2022, demonstrating that the BCH family of storage codes on triangle-free Cayley graphs achieves a unit rate. Furthermore, we generalize the construction of the BCH family and obtain more storage codes of unit rate on triangle-free graphs. We also compare the BCH family with the other known constructions by examining the rate of convergence of $1/(1-R(C_n))$ with respect to the length n, where $R(C_n)$ is the rate of code C_n. At last, we reveal a connection between the storage codes on triangle-free graphs and the Ramsey number $R(3,t)$, which leads to an upper bound for the rate of convergence of $1/(1-R(C_n))$.

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