

Graphs with prescribed edge-lengths: open problems and new results

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Abstract

The literature on graphs with prescribed edge-lengths is rich of attractive problems often linked with combinatorial topics of various kinds. A special attention has been devoted to a conjecture on Hamiltonian paths of the complete graph with a given list of edge-lengths, proposed by Buratti and then generalized in [1].

In [4] the authors proposed the following conjecture: a multiset L of n positive integers not exceeding n is the list of edge-lengths of a suitable near 1-factor F of the complete graph on $\{0, 1, \dots, 2n\}$ if and only if it contains at most $\frac{2n+1-d}{2}$ multiples of any divisor d of $2n+1$. The case n prime was already considered in [5], where a complete, but non constructive, solution is presented. Also the case in which the complete graph has an even number of vertices, and hence F is a 1-factor, has been investigated, see for instance [2, 3].

In this talk I will survey the results on these conjectures and I will focus on some related open problems.

References

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