New advances on graph families associated with graphicable algebras

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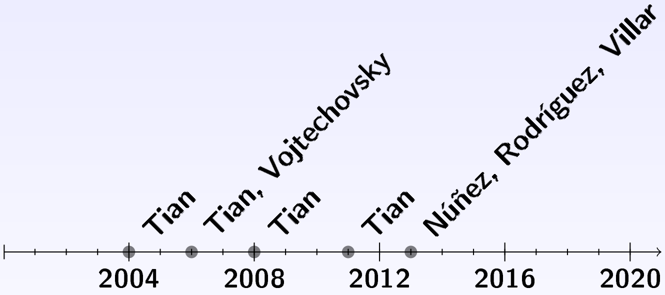

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Connection between evolution algebras and graphs

Path, cycle and complete graphicable algebras

Complete n-partite, friendship, star, snark, wheel and Petersen generalized graphicable algebras.

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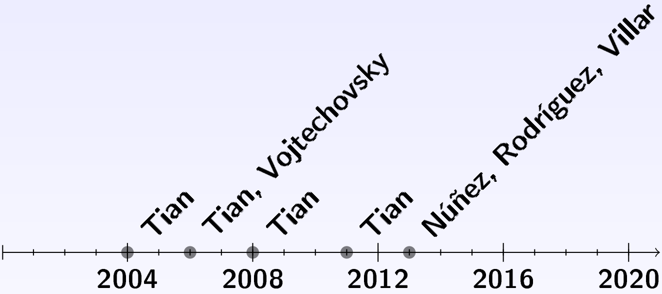

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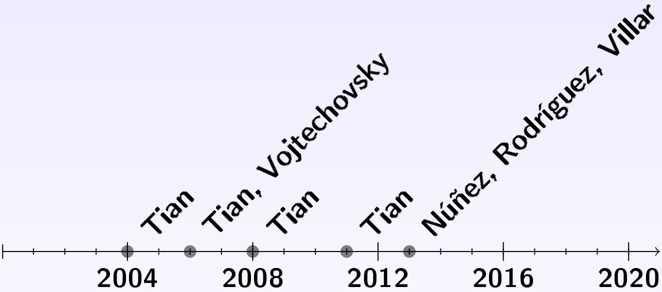

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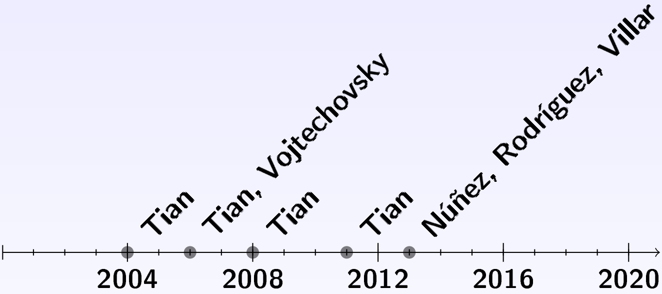

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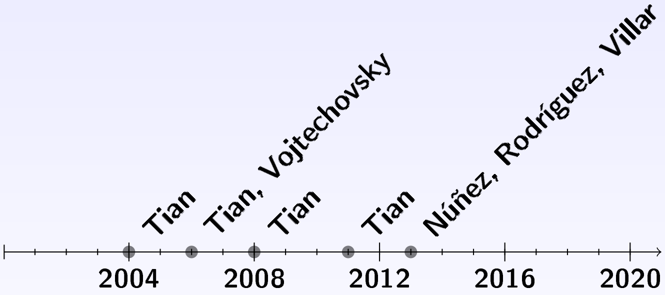

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Preliminaries on evolution algebras

Evolution algebra

Let $\mathcal{E} \equiv (\mathcal{E}, +, \cdot)$ be an algebra over a field $\mathbb{K}$. The algebra $\mathcal{E}$ will be called n-dimensional *evolution algebra* if we can find a basis $B = \{e_i\}_{i=1}^{n}$ of $\mathcal{E}$ such that

1) $e_i^2 = \sum_{k=1}^{n} c_{i,k}\, e_k$, $\forall\, 1 \leq i \leq n$; and
2) $e_i \cdot e_j = 0$, $\forall\, 1 \leq i \neq j \leq n$.

B is known as a *natural basis* of $\mathcal{E}$, $c_{i,k}$ are called the *structure constants* and $A = (c_{i,k})$ is named as the *structure matrix*. We will consider evolution algebras over the complex numbers field with natural basis B (also called set of generators).

Proposition

Evolution algebras are flexible and commutative, but they are not associative either power-associative.

Derived series

The **derived series** of an evolution algebra $\mathcal{E}$ is defined as follows:
$\mathcal{E}^{(1)} = \mathcal{E},\ \mathcal{E}^{(2)} = \mathcal{E} \cdot \mathcal{E},\ \ldots,\ \mathcal{E}^{(k)} = \mathcal{E}^{(k-1)} \cdot \mathcal{E}^{(k-1)},\ \ldots$

Solvable evolution algebra

$\mathcal{E}$ is called $(m-1)$-step **solvable** if there exists $m \in \mathbb{N}$ such that $\mathcal{E}^{(m)} = \{0\}$ and $\mathcal{E}^{(m-1)} \neq \{0\}$.
m is known as the *solvability index* or *solvindex* of $\mathcal{E}$.

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Perfect evolution algebra

The derived algebra of an evolution algebra $\mathcal{E}$ will be denoted by $D\mathcal{E} = \mathcal{E}^{(2)}$. An evolution algebra $\mathcal{E}$ is perfect if $\mathcal{E}$ and $D\mathcal{E}$ are isomorphic.

Graphicable algebra

An n-dimensional graphicable algebra is an evolution algebra where the structure constants are $c_{i,j} \in \{0,1\}, \forall 1 \leq i,j \leq n$ for a natural basis B.

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Preliminaries on Graph Theory

Graph

A **graph** is defined as a pair $G = (V, E)$, where V is a non-empty set known as the vertex-set (or node-set) and E is called the edge-set, that is determined by unordered pairs of elements from V.

Neighbours and neighbourhood

A pair of vertices in a graph G are *adjacent or neighbours* if they determine an edge on G. The *neighbourhood* of v, $N(v)$, are the neighbours of v. The *adjacency matrix* of G is a matrix $A = (a_{ij})$, where $a_{i,j} = 1$ if v_i and v_j are adjacent and $a_{i,j} = 0$ otherwise.

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Degree

The number of vertices adjacent to a vertex v is known as the *degree* of v.

Example (Types of graphs)

Regular graphs, Cubic graphs, Path graph, Cycle graph, Complete graph, Wheel graph, Complete n-partite graph, Friendship graph, n-cubes, fullerene, platonic solid Blanusa and *Pappus graph*

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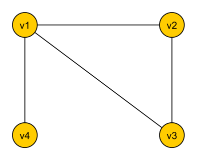
Graphicable algebra associated with a given graph

Let $G = (V, E)$ be a graph. Then, G can be associated with a graphicable algebra by considering the generators $V = \{e_1, e_2, \ldots, e_r\}$ and relations

$$e_i^2 = \sum_{e_k \in N(e_i)} e_k, \qquad e_i \cdot e_j = 0,\ i \neq j$$



This graph is associated with the graphicable algebra with $B = \{e_1, e_2, e_3, e_4\}$ and products $e_1^2 = e_2 + e_3 + e_4$, $e_2^2 = e_1 + e_3$, $e_3^2 = e_1 + e_2$, $e_4^2 = e_1$

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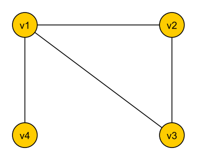

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What is the graph associated with the graphicable algebra with $B = \{e_1, e_2, e_3\}$ and $e_1^2 = e_2 + e_3,\ e_2^2 = e_1 + e_2,\ e_3^2 = e_2$?

S-Graphicable algebras

S-graphicable algebras are those which have a simple graph associated ($c_{i,j} = c_{j,i}$, $c_{i,i} = 0$, for $1 \leq i \neq j \leq n$).

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S-Graphicable algebras

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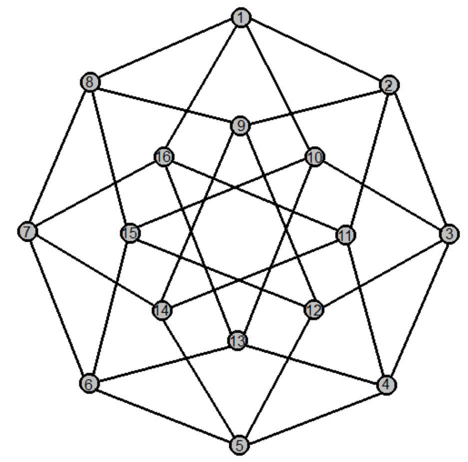
Hypercube graphicable algebra

The hypercube graphicable algebra, $A(Q_4)$, associated to the hypercube graph Q_4 has natural basis $\{e_1, \ldots, e_{16}\}$ and law

$$
\begin{aligned}
e_i^2 &= e_{i-1} + e_{i+1} + e_{i+7} + e_{i+9} \pmod{16}, && 1 \leq i \leq 8;\\
e_i^2 &= e_{i-9} + e_{i-7} + e_{i+3} + e_{i+5} \pmod{16}, && i = 10, 11;\\
e_i^2 &= e_{i-9} + e_{i-7} + e_{i-3} + e_{i+3} \pmod{16}, && i = 12, 13;\\
e_i^2 &= e_{i-9} + e_{i-7} + e_{i-5} + e_{i-3} \pmod{16}, && i = 14, 15;\\
e_i^2 &= e_{i-7} + e_{i-1} \pmod{8} + e_{i+3} + e_{i+5} \pmod{16}, && i = 9, 16.
\end{aligned}
$$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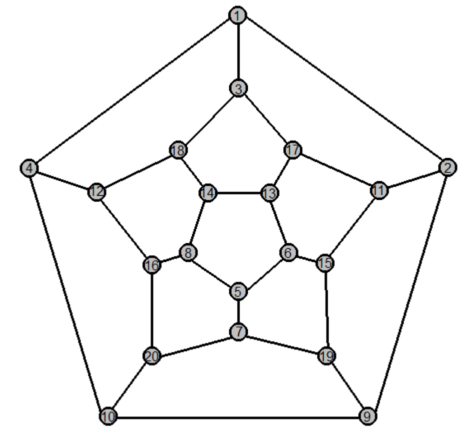
20-fullerene graphicable algebra

The 20-fullerene graphicable algebra, $A(F_{20})$, associated to the 20-fullerene, F_{20}, has natural basis $\{e_1, \ldots, e_{20}\}$ and law

$$\begin{array}{lll}
e_i^2 = & \sum_{j=1}^{3} e_{i+j}, & i = 1, 5; \\
e_i^2 = & e_{i-1-2\delta_{i,4}-2\delta_{i,8}} + e_{i+7-\delta_{i,4}-\delta_{i,8}} & \\
 & + e_{i+9-\delta_{i,4}-\delta_{i,8}}, & i = 2, 4, 6, 8; \\
e_i^2 = & e_{i-2} + e_{i+12+2\delta_{i,3}} + e_{i+13+2\delta_{i,3}}, & i = 3, 7; \\
e_i^2 = & e_{i-9+\delta_{i,12}+\delta_{i,16}} + e_{i+4} + e_{i+6-10\delta_{i,15}-\delta_{i,16}}, & i = 11, 12, 15, 16; \\
e_i^2 = & e_{i-7+\delta_{i,10}+\delta_{i,14}} + e_{i+9+\delta_{i,9}+4\delta_{i,14}+5\delta_{i,13}} & \\
 & + e_{i+10-6\delta_{i,13}-6\delta_{i,14}}, & i = 9, 10, 13, 14; \\
e_i^2 = & e_{3+4\delta_{i,19}+4\delta_{i,20}} + e_{i-6-4\delta_{i,19}-4\delta_{i,20}} + e_{i-4}, & i = 17, \ldots, 20.
\end{array}$$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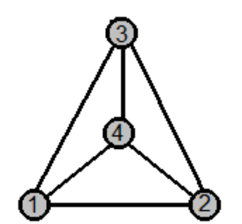
Platonic solid graphicable algebras

Tetrahedron graphicable algebra

The Tetrahedron graphicable algebra, $A(T)$, associated to the tetrahedron T has natural basis $\{e_1, \ldots, e_4\}$ and law

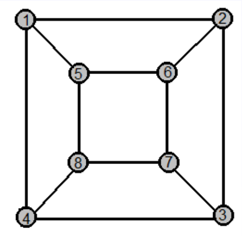
$$e_i^2 = \sum_{j=1, j\neq i}^{4} e_j, \text{ for } 1 \leq i \leq 4$$



Cube graphicable algebra

The Cube graphicable algebra, $A(Q_3)$, associated to the cube graph Q_3 has natural basis $\{e_1, \ldots, e_8\}$ and law

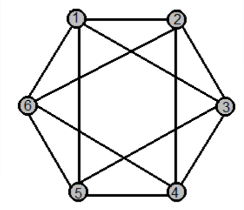
$$
\begin{aligned}
e_i^2 &= e_{i+1} + e_{i+3} + e_{i+4} \pmod 8, && \text{for } i = 1, 5;\\
e_i^2 &= e_{i-1} + e_{i+1} + e_{i+4} \pmod 8, && \text{for } i = 2, 3, 6, 7;\\
e_i^2 &= e_{i-3} + e_{i-1} + e_{i+4} \pmod 8, && \text{for } i = 4, 8.
\end{aligned}
$$



Octahedral graphicable algebra

The octahedral graphicable algebra, $A(O)$, associated to the octahedral graph O has natural basis $\{e_1, \ldots, e_6\}$ and law

$$e_i^2 = \sum_{j=1,\, i \neq j \neq i+3\, (mod\, 6)}^{6} e_j$$



Dodecahedral graphicable algebra

The dodecahedral graphicable algebra, $A(D)$, associated to the dodecahedral graph D has natural basis $\{e_1, \ldots, e_{20}\}$ and law

$$\begin{array}{ll} e_i^2 = e_{i+1} + e_{i-1}\,(\mathrm{mod}\,10) + e_{i+10}, & \forall\, 1 \leq i \leq 10 \\ e_i^2 = e_{i-10} + e_{\varphi(i)} + e_{\phi(i)}, & \text{for } 10 \leq i \leq 20, \end{array}$$

$$\text{where } \varphi(i) = \begin{cases} i+8 & if \quad i+8 \leq 20 \\ i-2 & if \quad i+8 > 20 \end{cases}$$

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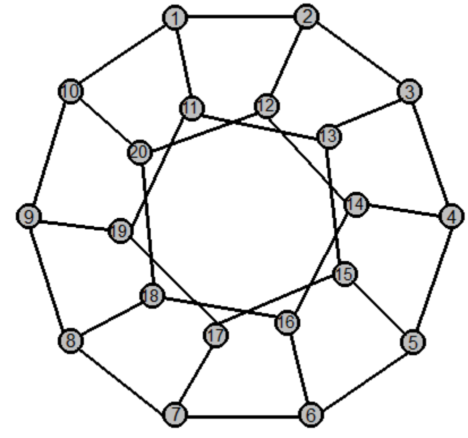
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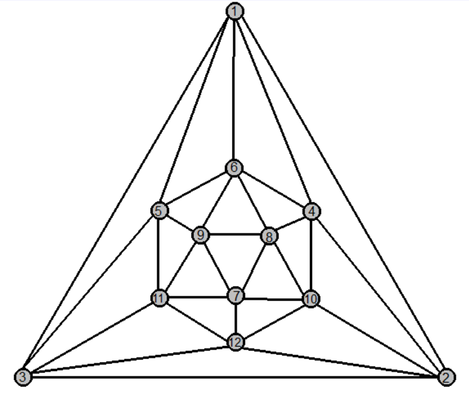
Icosahedral graphicable algebra

The icosahedral graphicable algebra, $A(I)$, associated to the icosahedral graph I has natural basis $\{e_1, \dots, e_{12}\}$ and law

$$\begin{aligned}
e_i^2 &= \sum_{j=1}^{5} e_{i+j}, && \text{for } i = 1, 7;\\
e_i^2 &= e_1 + e_{12} + e_{i+2} + e_{i+8} + e_{2+\delta_{2,i}}, && \text{for } i = 2, 3;\\
e_i^2 &= e_1 + e_6 + e_{i-2} + e_{i+4} + e_{i+6}, && \text{for } i = 4, 5;\\
e_i^2 &= e_6 + e_7 + e_{i+2} + e_{i-4} + e_{8+\delta_{8,i}}, && \text{for } i = 8, 9;\\
e_i^2 &= e_7 + e_{12} + e_{i-2} + e_{i-6} + e_{i-8}, && \text{for } i = 10, 11;\\
e_i^2 &= e_2 + e_3 + e_{i-5} + e_{i-2} + e_{i-1}, && \text{for } i = 6, 12.
\end{aligned}$$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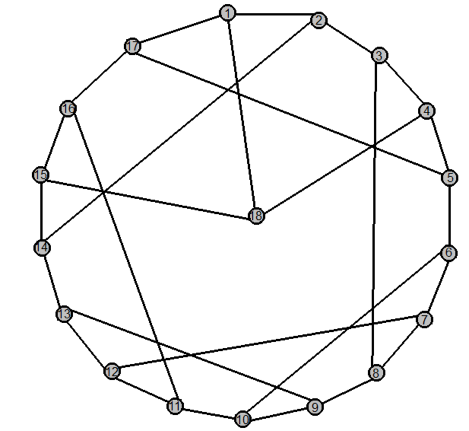
Blanusa graphicable algebra

The Blanusa graphicable algebra, $A(B)$, associated to the Blanusa graph B has natural basis $\{e_1, \ldots, e_{18}\}$ and law

$$
\begin{array}{ll}
e_i^2 = e_{i-1} + e_{i+1} + e_{i+12}, & \text{for } i = 2, 5; \\
e_i^2 = e_{i-1} + e_{i+1} + e_{i+5} \pmod{18}, & \text{for } i = 3, 7, 11; \\
e_i^2 = e_{i-1} + e_{i+1} + e_{i+14} \pmod{18}, & \text{for } i = 4, 10, 13; \\
e_i^2 = e_{i-1} + e_{i+1} + e_{i+4} \pmod{16}, & \text{for } i = 6, 9, 14; \\
e_i^2 = e_{i-1} + e_{i+1} + e_{i-5} \pmod{18}, & \text{for } i = 8, 12, 16; \\
e_i^2 = e_{i-1} + e_{i+1} + e_{i+3+3\delta_{i,14}+12\delta_{i,1}} \pmod{18}, & \text{for } i = 1, 14, 15; \\
e_i^2 = e_1 + e_{i-3+2\delta_{i,17}} + e_{i+4+2\delta i,17} \pmod{18}, & \text{for } i = 17, 18;
\end{array}
$$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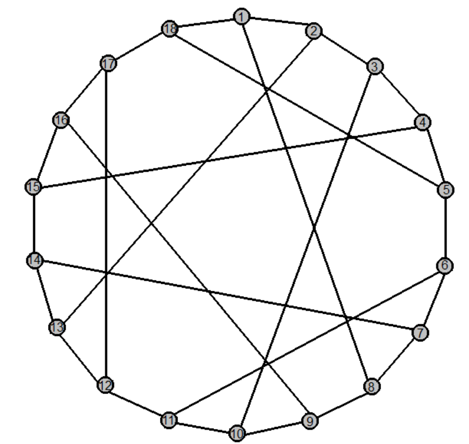
Pappus graphicable algebra

The Pappus graphicable algebra, $A(P)$, associated to the Pappus graph P has natural basis $\{e_1, \ldots, e_{18}\}$ and law

$$
\begin{aligned}
e_i^2 &= e_{i+1} + e_{i+7} + e_{i+17} \pmod{18}, && \text{for } i = 1, 3, 7, 9, 13, 15;\\
e_i^2 &= e_{i-1} + e_{i+1} + e_{i+11} \pmod{18}, && \text{for } i = 2, 4, 8, 10, 14, 16;\\
e_i^2 &= e_{i-1} + e_{i+1} + e_{i+13} \pmod{18}, && \text{for } i = 5, 11, 17;\\
e_i^2 &= e_{i-1} + e_{i+1} + e_{i+5} \pmod{18}, && \text{for } i = 6, 12, 18.
\end{aligned}
$$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Path, cycle, n-partite, friendship, complete and wheel graphs

- **The graphicable path algebra with n vertices, $A(P_n)$, is perfect if n is even and non-solvable if n is odd.**
- The graphicable cycle algebra with n vertices, $A(C_n)$, is non-solvable if $n = 4$ and it is perfect otherwise.
- The complete n-partite graphicable algebra $A(K_{\alpha_1,\alpha_2,\ldots,\alpha_n})$ with $\alpha_1 + \alpha_2 + \ldots + \alpha_n$ vertices is non-solvable.
- The graphicable friendship algebra with $2n + 1$ vertices, $A(F_n)$, is perfect.
- The graphicable complete algebra with n vertices, $A(K_n)$, is perfect.
- The graphicable wheel algebra with n vertices, $A(W_n)$, is non-solvable if $n = 5$ and it is perfect otherwise.

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- The graphicable cycle algebra with n vertices, $A(C_n)$, is non-solvable if $n = 4$ and it is perfect otherwise.
- The complete n-partite graphicable algebra $A(K_{\alpha_1,\alpha_2,\ldots,\alpha_n})$ with $\alpha_1 + \alpha_2 + \ldots + \alpha_n$ vertices is non-solvable.
- The graphicable friendship algebra with $2n + 1$ vertices, $A(F_n)$, is perfect.
- The graphicable complete algebra with n vertices, $A(K_n)$, is perfect.
- The graphicable wheel algebra with n vertices, $A(W_n)$, is non-solvable if $n = 5$ and it is perfect otherwise.

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Tetrahedron, cube, octahedral, dodecahedral, icosahedral and 20-fullerene graphs

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Subprocedure prod

```
> restart: with(LinearAlgebra): with(GraphTheory): with(ListTools):
> assign(dim,...):
> prod:=proc(i)
> if i=... then ...;
> elif ... then ...;
> ...
> elif ... then ...;
> else 0; fi;
> end proc;
```

Subprocedure Sgraphicable

```
> Sgraphicable:=proc(n)
> local L;
> L:=[];
> for i from 1 to n do
> L:=[op(L),coeff(prod(i),e[i])];
> for j from i+1 to n do
> L:=[op(L),coeff(prod(i),e[j])-coeff(prod(j),e[i])];
> od;
> od;
> L:=MakeUnique(L);
> if L=[0] then print(The algebra is S-graphicable)
> else print(The algebra is not S-graphicable)
> fi;
> end proc:
```

Subprocedure drawingraph

```
> drawingraph:=proc(n)
> local V,E;
> V:=[seq(i,i=1..n)];
> E:=;
> for i from 1 to n do
> if prod(i)<>0 then
> for j from 1 to n do
> if j<>i then
> if coeff(prod(i),e[j])<>0 then
> E:=op(E),i,j;
> fi;
> fi;
> od;
> fi;
> od;
> DrawGraph(Graph(V,E));
> end proc:
```

We consider the evolution algebra $\mathcal{E}$ with dimension 5, natural basis $\{e_i\}_{i=1}^5$ and law

$$e_1^2 = 2e_2 + e_3,\ e_3^2 = 2e_1 - e_3 + 2e_5,\ e_4^2 = e_1,\ e_5^2 = 3e_2 - e_3$$

Compleing subprocedure prod

```
> restart: with(LinearAlgebra): with(GraphTheory): with(ListTools):
> assign(dim,5):
> prod:=proc(i)
> if i=1 then 2*e[2]+e[3];
> elif i=3 then 2*e[1]-e[3]+2*e[5];
> elif i=4 then e[1];
> elif i=5 then 3*e[2]-e[3];
> else 0; fi;
```

Executing subprocedure Sgraphicable

```
> Sgraphicable(dim); The algebra is not S-graphicable
```

We consider the evolution algebra $\mathcal{E}$ with dimension 5, natural basis $\{e_i\}_{i=1}^{5}$ and law

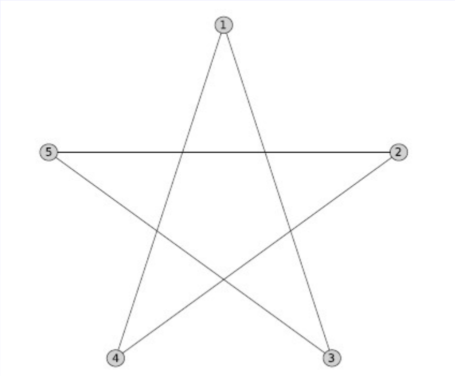
$$e_1^2 = e_3 + e_4, \; e_2^2 = e_4 + e_5, \; e_3^2 = e_1 + e_5, \; e_4^2 = e_1 + e_2, \; e_5^2 = e_2 + e_3$$

Completing subprocedure prod

```
> restart: with(LinearAlgebra): with(GraphTheory): with(ListTools):
> assign(dim,5):
> prod:=proc(i)
> if i=1 then e[3]+e[4];
> elif i=2 then e[4]+e[5];
> elif i=3 then e[1]+e[5];
> elif i=4 then e[1]+e[2];
> elif i=5 then e[2]+e[3];
> else 0; fi;
```

Executing subprocedure Sgraphicable

```
> Sgraphicable(dim); The algebra is S-graphicable
> drawingraph(dim);
```


1
2
3
4
5

Open problems

1. Using the graphicable algebras $A(P_n)$, $A(C_n)$, $A(F_n)$, $A(K_n)$, $A(W_n)$, $A(T)$, $A(Q_3)$, $A(I)$ to compute perfect subalgebras of evolution algebras.
2. Introducing other graph notions attached to graphicable algebras such as planar graphicable algebra.
3. Studying the type of graphicable algebra under graph operations such as union, intersection, join, products, etc.
4. Translating general concepts of graph theory such us colorability and connectivity to the language of graphicable algebras.

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Thank you

Thank you very much!