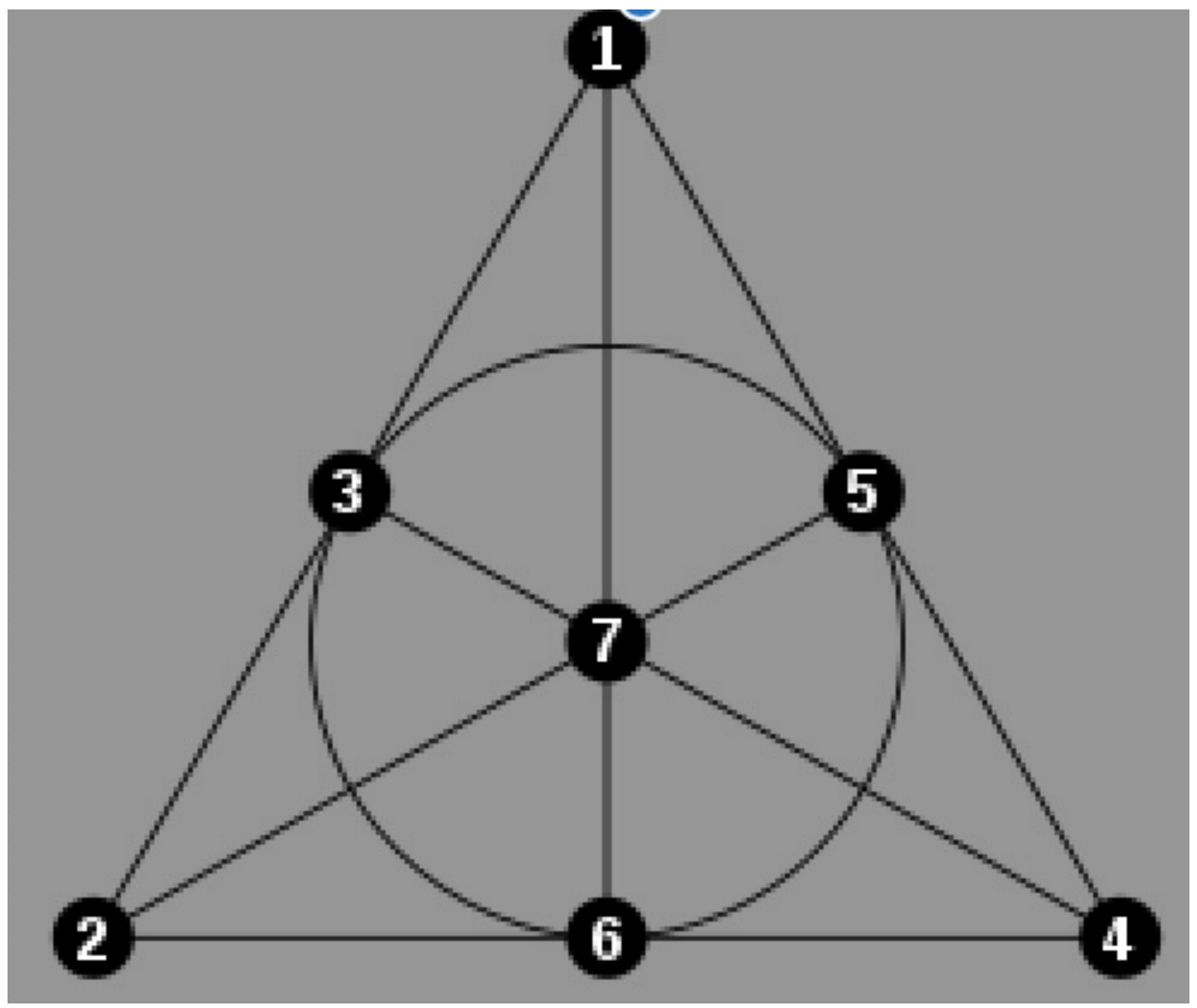
New Spence Difference Sets

Joint with J. Polhill, K. Smith, E. Swartz, and J. Webster

Why should you care?

Automorphism groups of combinatorial objects

Design Theorists Coat of Arms (according to Eric Lander)



Very brief history

1. Cyclic (Singer, Paley, twin primes) 1930s-1960s
2.

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2. Non-cyclic (Hadamard, McFarland, Spence, Davis-Jedwab, Chen) 1960s-present

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K a group of coefficient; $D = k_1H_1 \cup k_2H_2 \cup \ldots \cup k_dH_d$

Spence Difference Sets

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- **complement one hyperplane, say H_1^c**
- **K any group (including nonabelian)**

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[G:V] is a prime power

Not totally accurate……

$$H_i \longrightarrow k_i H_i k_i^{-1}$$

Smallest case (36,15,6)

Exhaustively studied by Smith and Webster

Next size (351,126,45)

Different approach: Transfer method

Key idea here: choose the k_i so that the full automorphism group is large

Upshot: lots of new Spence DSs in nonabelian groups

Intriguing example: Sylow 3-subgroup is NOT elementary abelian (and is in fact nonabelian)

Future Directions

1. Conjecture: if G is a Spence group with a normal subgroup V, then G has a Spence DS
2. Exhaustively do the (351,126,45) case
3. Determine the number of nonisomorphic designs
4. Exploit the transfer method even more!

Grazie Mille!