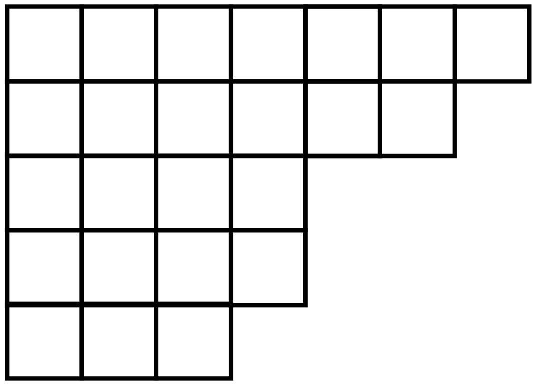
On Latin Young Diagrams

Daniel Kotlar

Tel-Hai College

Joint work with Ron Aharoni, Eli Berger and He Guo

Young diagram

$Y =$ 

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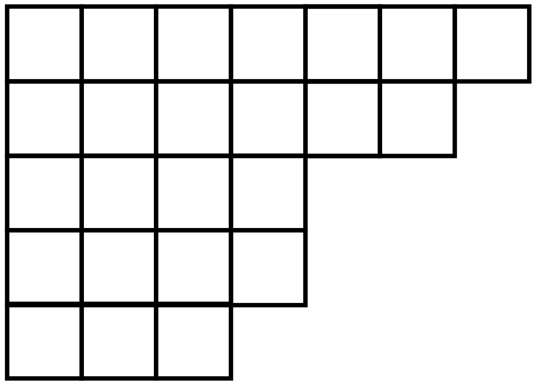
$Y =$ 

m rows: $r_1, r_2, \ldots, r_m$

row lengths: $a_1, a_2, \ldots, a_m$ respectively

a_1 columns: $c_1, c_2, \ldots, c_{a_1}$

$$|Y| = \sum_{i=1}^{m} a_i$$

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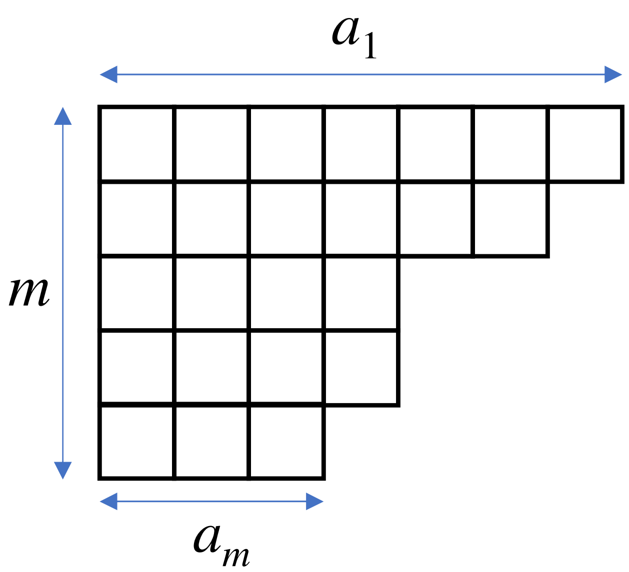

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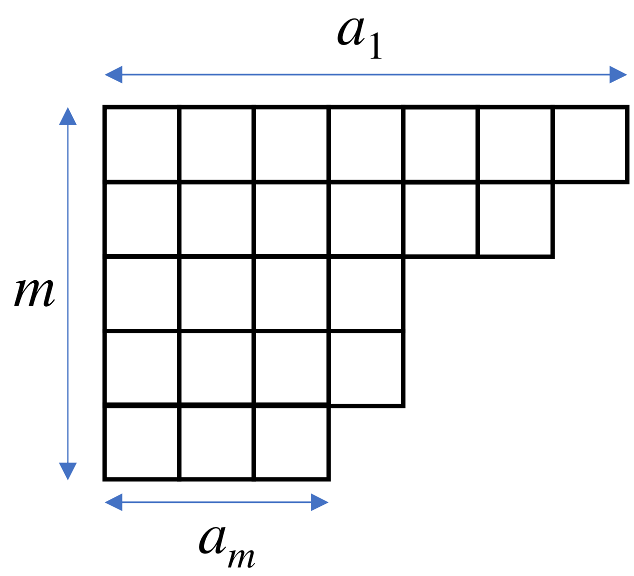
$Y =$ 

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Is it possible to place in each row r_i the numbers $1, 2, \ldots, a_i$ so that the entries in each column are distinct?

* T. Y. Chow, C. K. Fan, M. X. Goemans, and J. Vondrak. Wide partitions, Latin tableaux, and Rota's basis conjecture. *Adv. in Appl. Math.* **31** (2003), 334–358.

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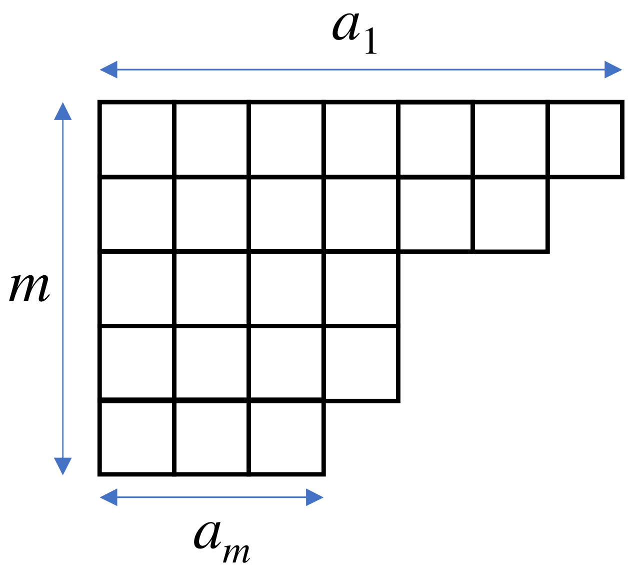

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If the answer is *Yes* we say that Y is **Latin**

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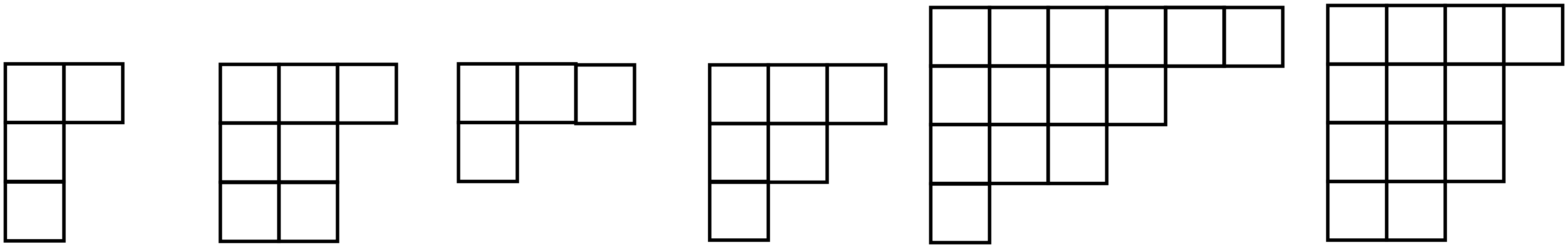

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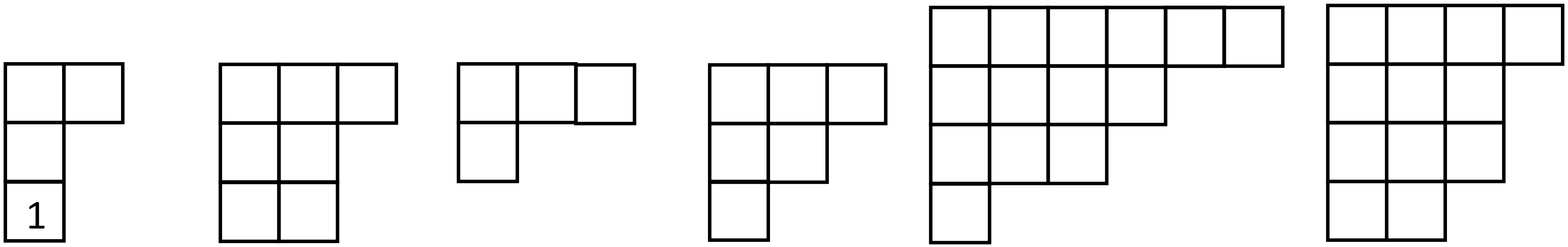

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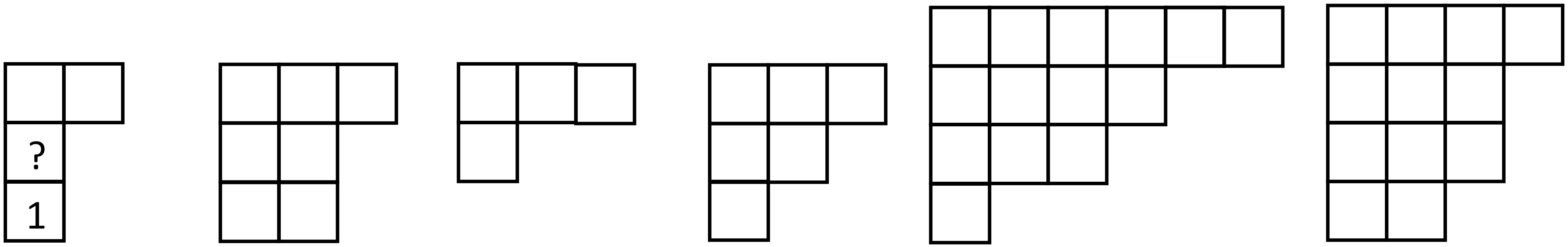

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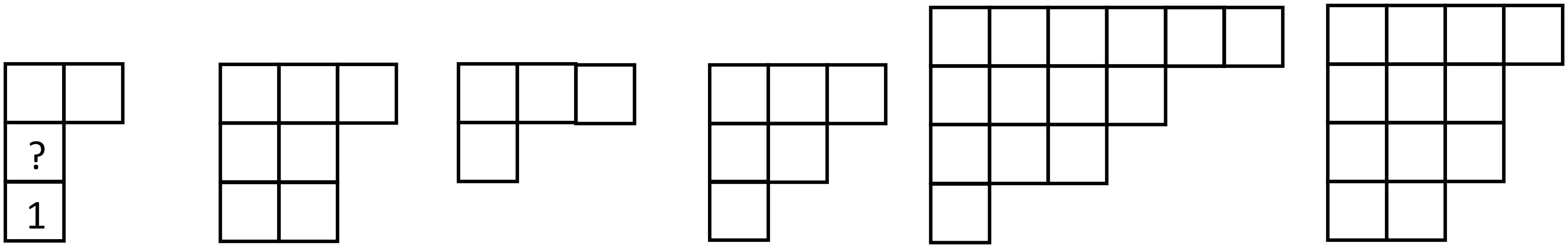

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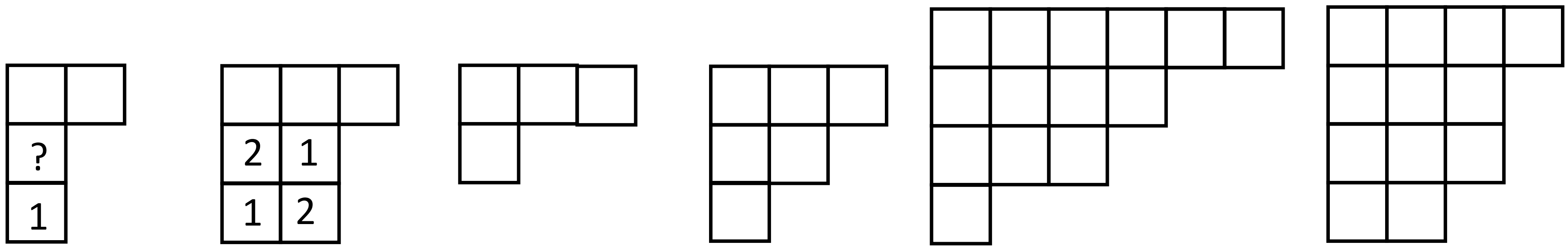

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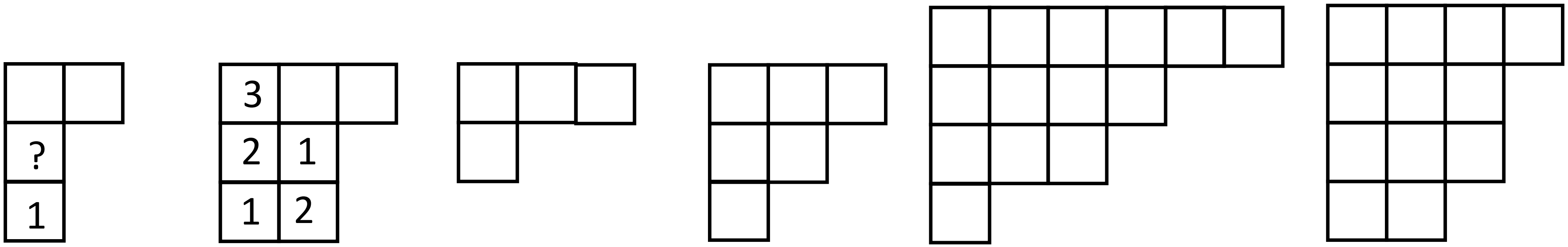

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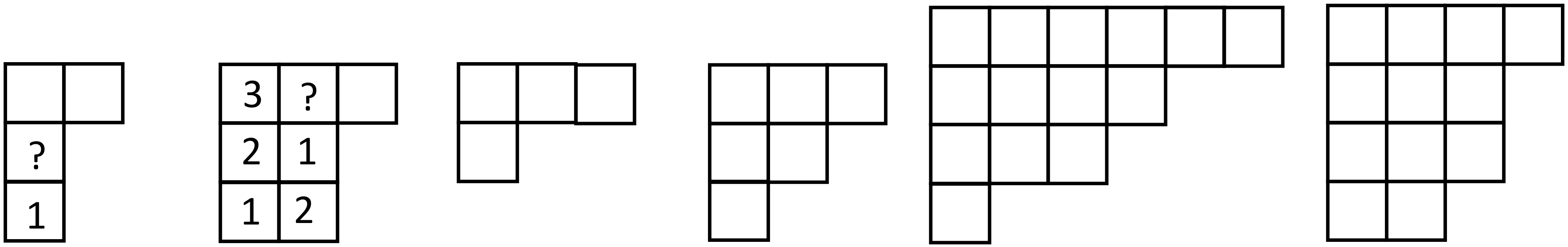

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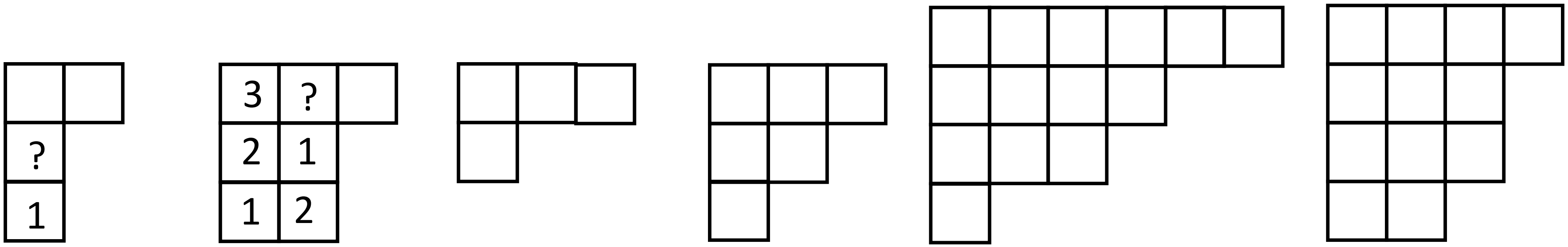

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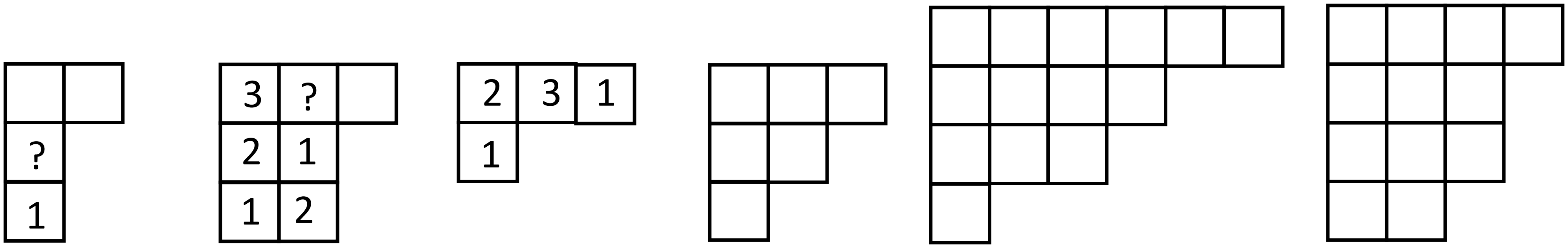

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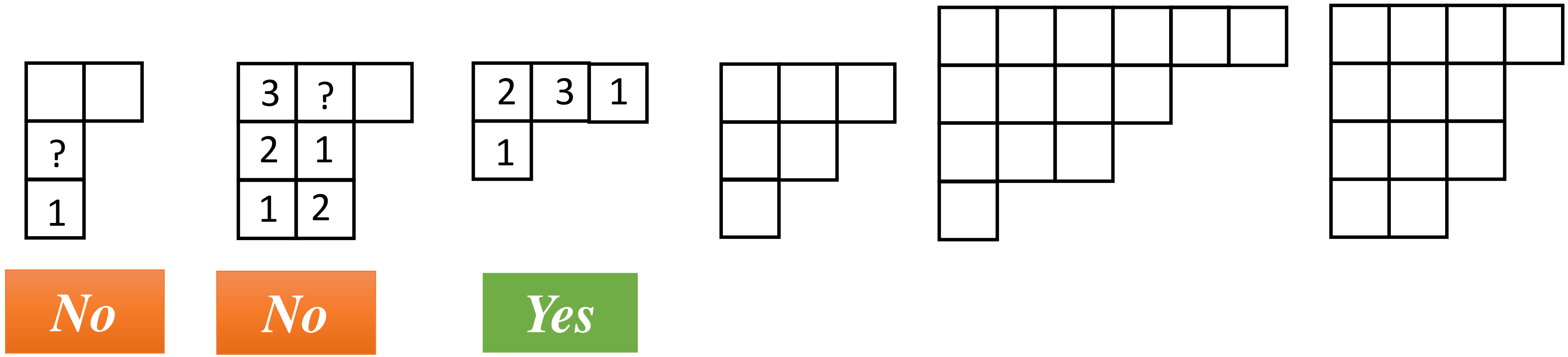

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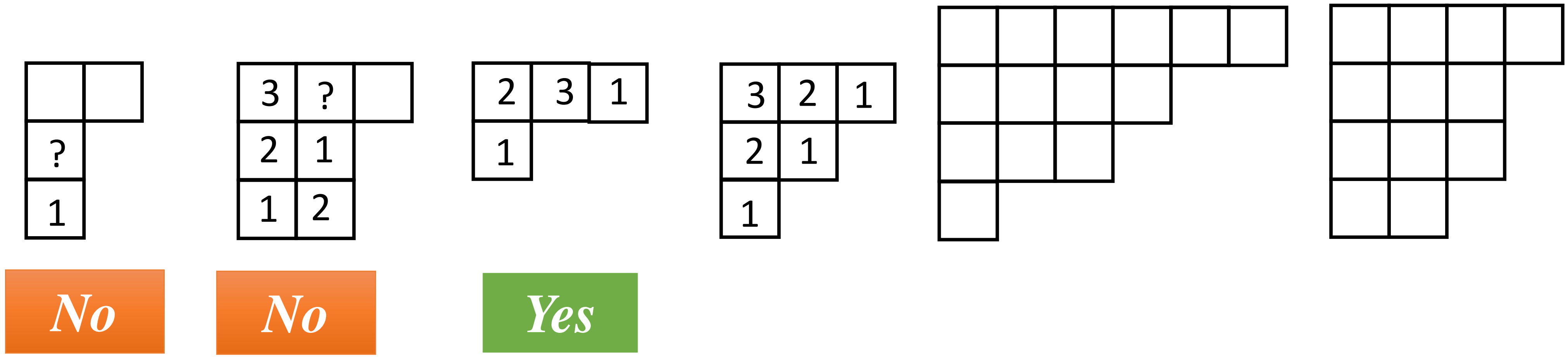

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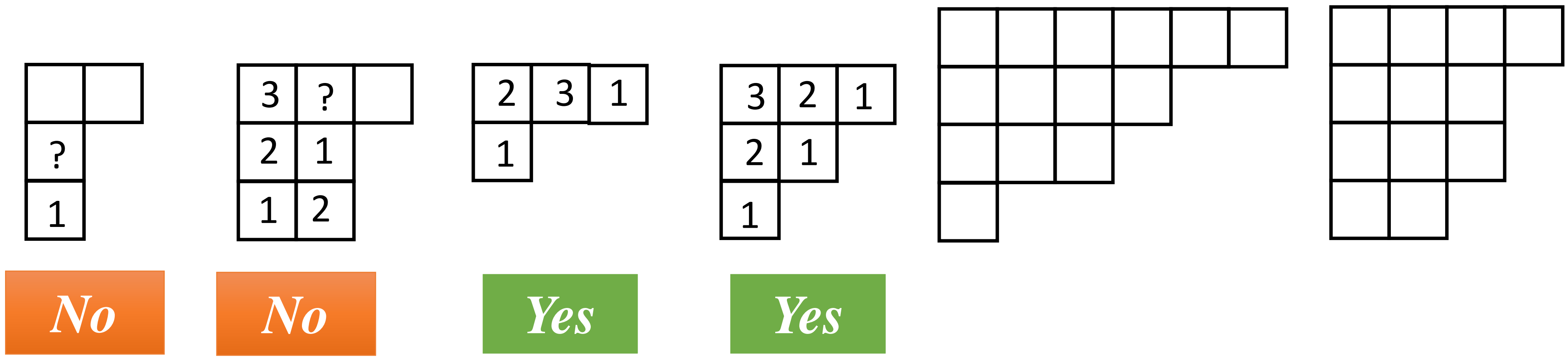

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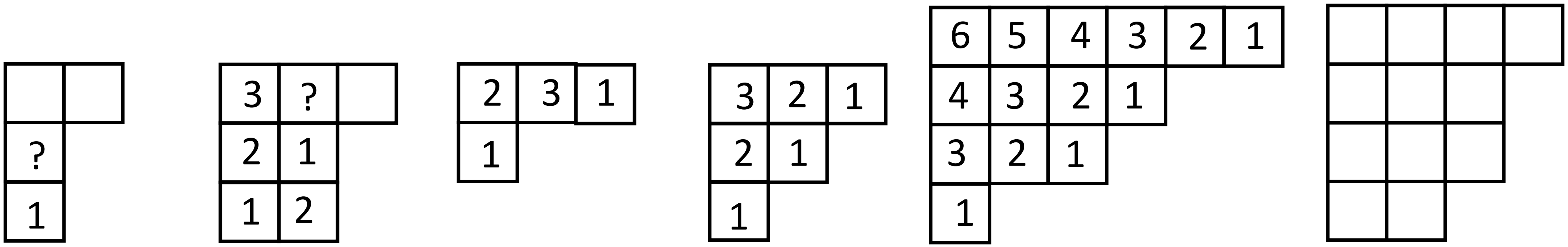

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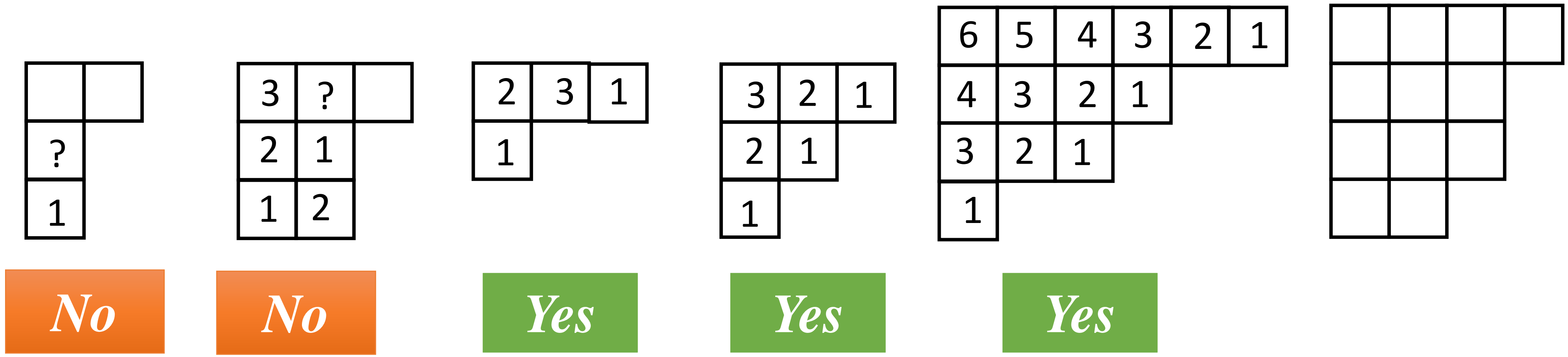

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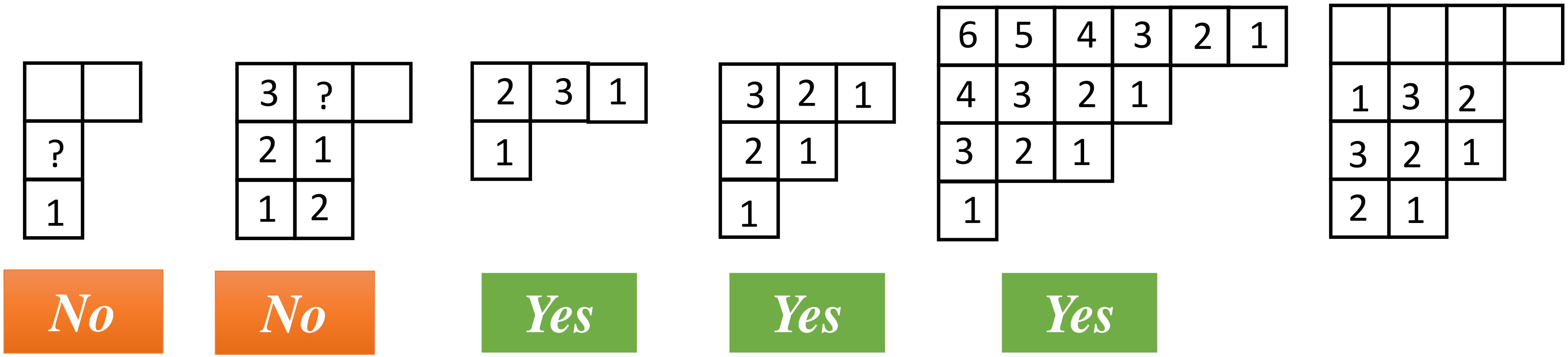

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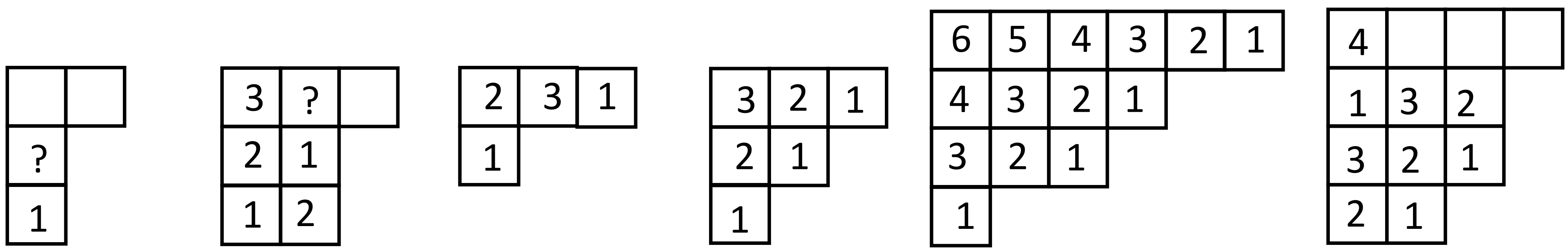

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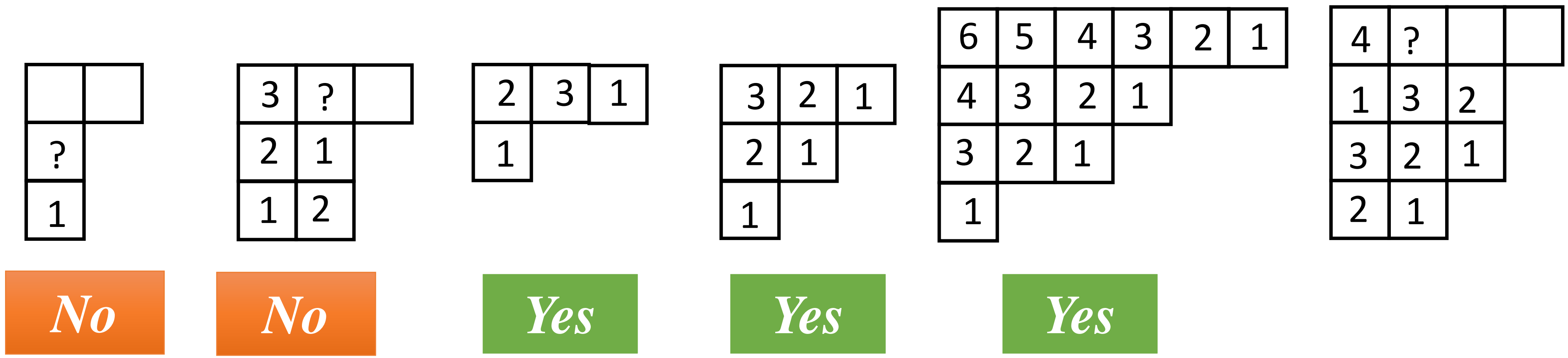

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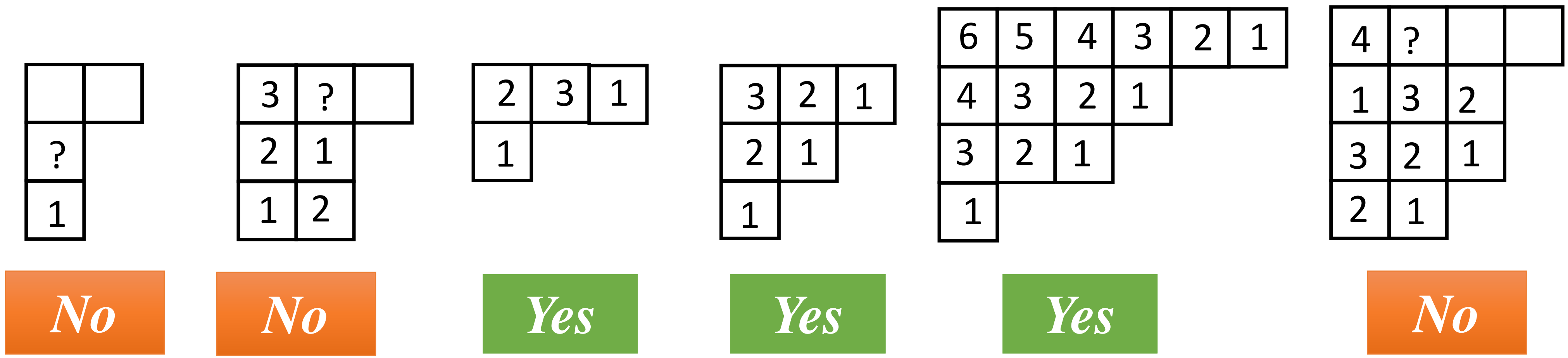

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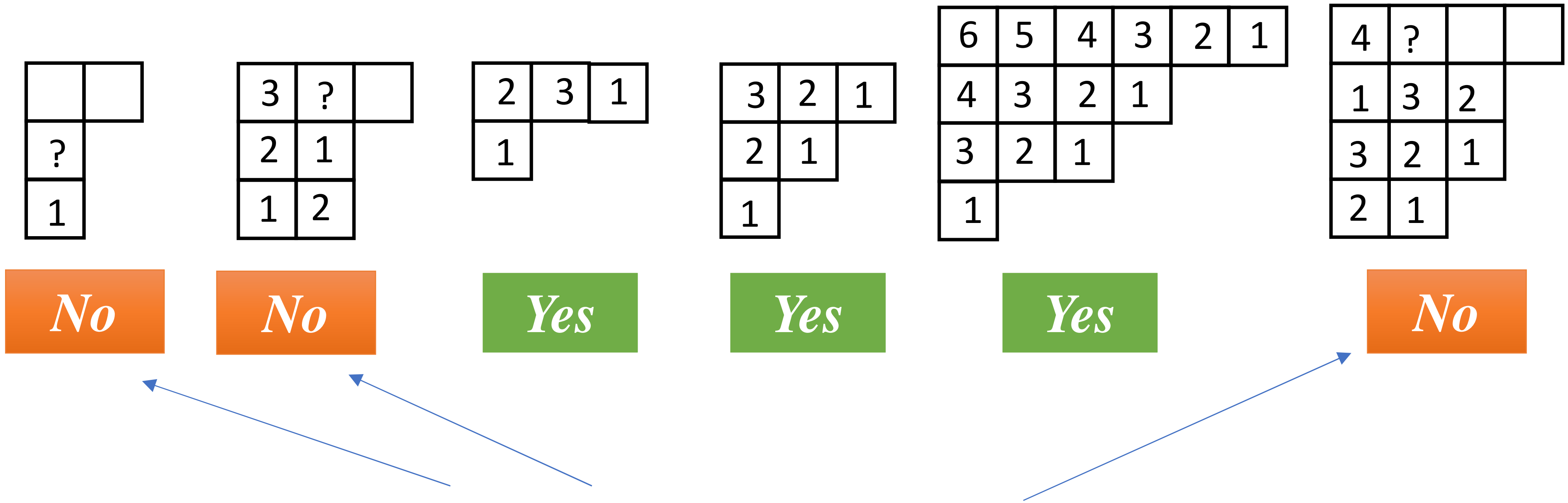

These diagrams are "too narrow"

Wide diagrams

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$$A = (a_1, a_2, \ldots), \;\; a_1 + a_2 + \cdots = n, \qquad a_1 \geq a_2 \geq \cdots$$

$$B = (b_1, b_2, \ldots), \;\; b_1 + b_2 + \cdots = n, \qquad b_1 \geq b_2 \geq \cdots$$

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A Young diagram Y is **wide** if every sub-diagram Z formed by a subset of the rows of Y dominates Z', the conjugate of Z.

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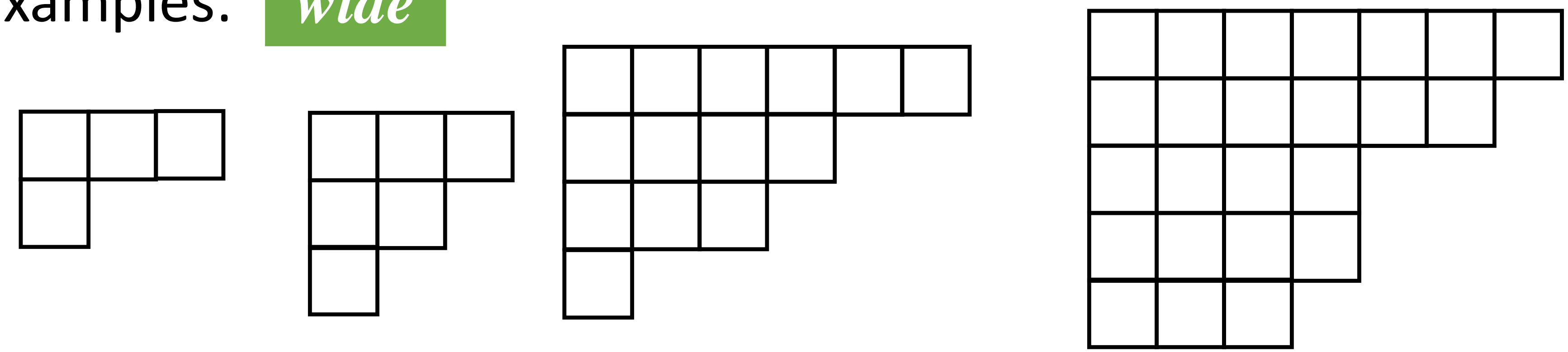
Examples: *wide*



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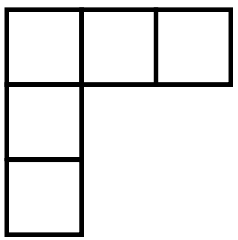
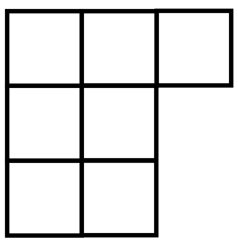
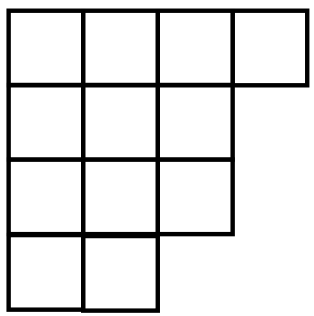
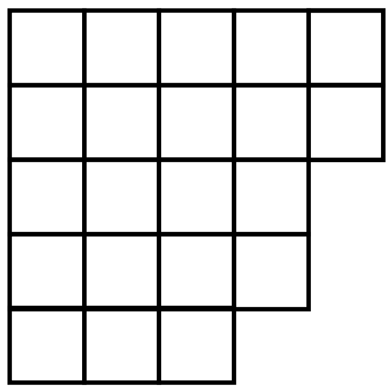
Examples: ***not wide***









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WPC

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Theorem:
If the wide partition conjecture holds for self-conjugate wide diagrams, then it holds for all wide diagrams.

The 3-hypergraph $H(Y)$

To a Young diagram Y we assign a tripartite 3-hypergraph $H(Y)$ as follows:

Sides:

$$R = R(Y) = \{r_1, r_2, \dots, r_m\} \quad \text{(the rows)}$$

$$C = C(Y) = \{c_1, c_2, \dots, c_{a_1}\} \quad \text{(the columns)}$$

$$S = S(Y) = \{s_1, s_2, \dots, s_{a_1}\} \quad \text{(the symbols)}$$

Edges:

$$E(H(Y)) = \{r_i c_j s_k \mid 1 \leq i \leq m, c_j \in C, s_k \in S, 1 \leq j, k \leq a_i\}$$

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A filling of Y with symbols from S corresponds to a set of $|Y|$ edges in $H(Y)$.

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A Latin filling of Y with symbols from S corresponds to a set of $|Y|$ edges in $H(Y)$ no two of which share more than one vertex.

The kth matching number

Definition: (Aharoni and Zerbib*, 2020)
a k-matching in a hypergraph H is a subset of $\mathrm{E}(H)$ in which every two edges share fewer than k vertices.

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WPC is equivalent to:

Wide Partition Conjecture (hypergraph version):
If Y is wide, then $\nu^{(2)}\big(H(Y)\big) = |Y|$

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Easy to see:

$$\tau^{(k)}(H) \geq \nu^{(k)}(H)$$

(Given a k-matching M of maximal size $\nu^{(k)}(H)$, we need at least $|M|$ k-sets of edges to cover its members.)

A weak version of WPC

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If a Young diagram Y is wide, then $\tau^{(2)}(H(Y)) = |Y|$

(This is weaker than WPC, since $\tau^{(2)}(H) \geq \nu^{(2)}(H)$)

The easier (but not trivial) direction:

Theorem 2 (A,B,G,K 2023):
If $\tau^{(2)}(H(Y)) = |Y|$, then the Young diagram Y is wide

This strengthens Observation 1:

Y is Latin $\Rightarrow$ $\nu^{(2)}(H(Y)) = |Y|$

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A fractional version

A fractional 2-matching in a hypergraph H is a function $f: E(H) \to \mathbb{R}_{\geq 0}$ subject to the constraint

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A fractional 2-cover of a hypergraph H is a function $g\colon \binom{V(H)}{2} \to \mathbb{R}_{\geq 0}$ subject to the constraint

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By the definition and LP duality:

$$\tau^{(k)}(H) \geq \tau^{(k)*}(H) = \nu^{(k)*}(H) \geq \nu^{(k)}(H)$$

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Conjecture:
If Y is a wide Young diagram, then Y is fractionally Latin

$\nu^{(2)}$ vs. $\tau^{(2)}$

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For a Young diagram Y $\tau\big(H(Y)\big) = \nu\,(H(Y))$

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Is it true that $\tau^{(2)}\big(H(Y)\big) = \nu^{(2)}\,(H(Y))$ for every Young diagram Y?

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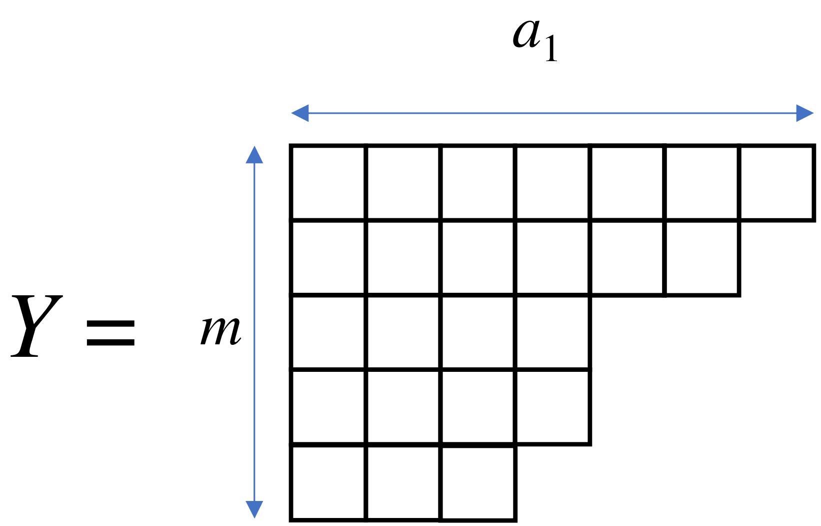
If the Wide Partition Conjecture is true, then the answer is YES

M-tableaux



m rows: $r_1, r_2, \ldots, r_m$

row lengths: $a_1, a_2, \ldots, a_m$ respectively

a_1 columns: $c_1, c_2, \ldots, c_{a_1}$

M a matroid

An M-tableau is a Young diagram with an element of M in each of its cells

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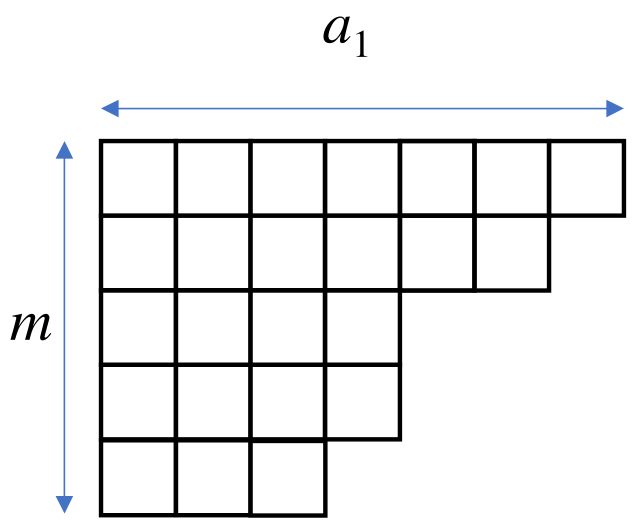
$Y =$



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Question (Chow, Fan, Goemans, Vondrak 2003*):
Given an M-tableau such that the elements in each row are independent, is it possible to rearrange the elements in each row so that the elements in each column are independent?

* T. Y. Chow, C. K. Fan, M. X. Goemans, and J. Vondrak. Wide partitions, Latin tableaux, and Rota's basis conjecture. *Adv. in Appl. Math.* **31** (2003), 334–358.

Generalized Rota's Basis Conjecture

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If all the rows are of size $\mathrm{rank}(M)$ and $m \leq \mathrm{rank}(M)$, then the answer is Yes

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Generalized WPC (Chow, Fan, Goemans, Vondrak 2003):
The answer is Yes if and only if Y is wide

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