Algebraically-informed deep networks for associative evolution algebras

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Goals

- Obtaining new results on the study of associative evolution algebras, mainly related with their classifications.
- Using algebraically-informed deep networks to verify some results already obtained theoretically by us.
- Seeing the possibility of applying this technique to obtain new results which confirm our own conjectures.

Index

Evolution algebras

Regarding evolution algebras, Manuel Ceballos already dealt with evolution algebras last Tuesday in his talk at this event, although indirectly. He talked about graphicable evolution algebras, which constitute a particular case of them.

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Evolution algebras

Definition: Algebra

Vector space $(E,+)$ over a field $\mathbb{K}$ endowed with a bilinear product $(\cdot)$

Definition: Evolution Algebra

Algebra $(E,+,\cdot)$ such that there exists a basis (*natural basis*) $\mathcal{B}=\{e_i : i\in\Lambda\}$ with $e_i\cdot e_j=0$, if $i\neq j$.

$$e_j^2 = e_j\cdot e_j = \sum_{i\in\Lambda} a_{ij}e_i \longrightarrow A=(a_{ij})$$ Structure matrix

Evolution algebras are commutative, but they are not associative in general. The last part of this result can be seen in the following example: If we consider the evolution algebra defined by multiplication laws:

$$e_1^2 = e_2^2 = e_2,$$

then:

$$(e_1\cdot e_1)\cdot e_2 = e_2\cdot e_2 = e_2 \neq 0 = e_1\cdot(e_1\cdot e_2).$$

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Evolution algebras

Definition: non-degenerate and degenerate evolution algebras

An evolution algebra is said to be *non-degenerate* if $e_j^2 \neq 0$, for all $j \in \Lambda$. On the contrary, the algebra is called *degenerate*.

It is easy to know if an evolution algebra is degenerate or not, seeing its structure matrix, since, according to the previous definition, in the structure matrix of a non-degenerate evolution algebras there must be at least one column formed by zeros

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 8 & 2 \end{pmatrix} \qquad \begin{pmatrix} 1 & 7 & 0 \\ 6 & 3 & 0 \\ 2 & 7 & 0 \end{pmatrix}$$

Non-degenerate | Degenerate

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Evolution algebras

Definition: Evolution operator

Endomorphism $L : E \to E$ which maps each generator into its square, that is

$$L(e_j) = e_j^2 = \sum_{i \in \Lambda} a_{ij} e_i$$

The matrix representation of the evolution operator with respect to the basis $\mathcal{B}$ is the structure matrix A.

What is our work on evolution algebras about?

As we said at the beginning of the talk, evolution algebras are not associative in general, but some of them are.

We have tried of classifying several types of complex evolution algebras: associative evolution algebras and evolution algebras in general whose evolution operator is either a derivation or an homomorphism of algebras. Our results have been the following

- Classification of **associative evolution algebras**:

  Complete classification.

  This classification has been published by us in the reference:

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Finally, with respect to the classification of complex evolution algebras in general, whose evolution operator is an homomorphism of algebras, this one is giving us some problems:

- We have obtained only **Classifications for rank** $n, n-1, n-2, 1$ of the structure matrix of the algebra.

For the rest of ranks, these classification are very complicated to obtain because the procedure involves equations very complex. In any case, we have really obtained necessary conditions for that rest of ranks, although not the complete classifications.

These results can be checked in the reference

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Algebraically-informed deep networks (AIDN)

So, we thought that a possible solution to advance in getting these classifications in the rest of ranks could be the use of neural networks, particularly AIDN.

Definition: Presentation

A presentation in AIDN is a system formed by a pair $\langle S \mid R \rangle$, where $S = \{s_i\}_{i=1}^{n}$ is a set of formal symbols (generators) and $R = \{r_i\}_{i=1}^{k}$ a formal set of equations (relations) that these generators satisfy.

Objective

Finding neural networks $\{f_i(x; \theta_i)\}_{i=1}^{n}$, where $\theta_i \in \mathbb{R}^{k_i}$ is the parameter vector of the network f_i, such that these neural networks correspond to the generators of S and satisfy the same relations of R.

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Algebraically-informed deep networks (AIDN)

AIDN algorithm

The AIDN algorithm finds the weights $\{\theta_i\}_{i=1}^n$ of the networks $\{f_i(x;\theta_i)\}_{i=1}^n$ by minimizing the loss function

$$\mathcal{L}(f_1,\dots,f_n)=\sum_{i=1}^{k} \parallel \mathcal{F}(r_i) \parallel_2^2,$$

where $\mathcal{F}(r_i)$ is the relation r_i written in terms of the networks $\{f_i(x;\theta_i)\}$.

AIDN for associative evolution algebras

With respect to associative evolution algebras and previously to use AIDN tecnique, we had already proved the following theoretical result on them:

Theorem

Let E be a non-degenerate evolution algebra. Then, the following assertions are equivalent

- E is associative.
- $e_i \cdot e_j^2 = 0$, for all $i \neq j$.
- The structure matrix is a diagonal matrix with nonzero diagonal elements.

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Then, what we wanted to do was to verify these results using the neural networks technique. To do this, we experimented with this

Presentation of an associative evolution algebra

- Generators: $\{e_i\}_{i=1}^{n}$
- Relations: $e_i \cdot e_j^2 = 0$, for all $i \neq j$ (one of the conditions of the theorem).
  $e_j^2 \neq 0$, for all j.

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AIDN for associative evolution algebras

This Table shows that the previous algorithm runs well for associative evolution algebras of different dimensions because, as we expected, matrices obtained are diagonal.

Dimension	Structure matrix
2	$\begin{pmatrix} 145.22 & 0 \\ 0 & -143.69 \end{pmatrix}$
3	$\begin{pmatrix} 19.94 & 0 & 0 \\ 0 & -20.81 & 0 \\ 0 & 0 & 20.68 \end{pmatrix}$
4	$\begin{pmatrix} -23 & 0 & 0 & 0 \\ 0 & -23.93 & 0 & 0 \\ 0 & 0 & -22.74 & 0 \\ 0 & 0 & 0 & -22.27 \end{pmatrix}$

Table 1: Examples of structure matrices of evolution algebras of different dimensions obtained after training AIDN. Loss values obtained were from order 10^{-4} to 10^{-5}.

AIDN for associative evolution algebras

This graphic represents the model loss with respect to the number of epochs (let me recall that an *epoch* is an iteration in the algorithm, which is used to minimize the loss function).

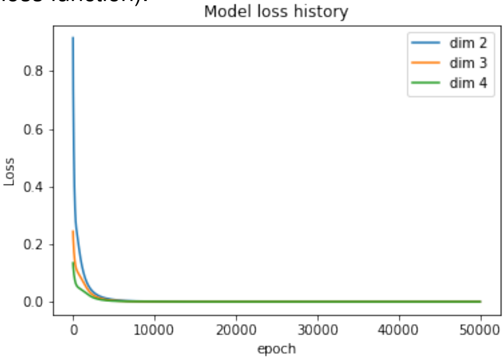

Figure 1: Neural networks were trained for 50000 epochs with a learning rate of 10^{-3}. Note that the evolution of the loss function during the training algorithm was similar in the three cases.

Future work

On the one hand, we have verified that the AIDN algorithm works well for associative evolution algebras.

On the other, we have obtained several theoretical results for evolution algebras in general.

Therefore, two possible open problems could be approached

- Check if AIDN also runs for other families of evolution algebras for which we know the complete classification, for instance, those in which the evolution operator is a derivation.

- Use AIDN to obtain evolution algebras whose evolution operator is a homomorphism, in order to obtain the full classification of these algebras.

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