Hadamard 2025

Sevilla, 26-30 May 2025

In honor of the 60-th birthday of Dane Flannery (University of Galway, Ireland) and the 65-th birthday of Robert Craigen (University of Manitoba, Canada).

DTHMA25

INVITED SPEAKERS

Ingemar Bengtsson (Stockholm University, Sweden)
Dean Crnkovic (University of Rijeka, Croatia)
Giora Dula (Netanya College, Israel)
Ronan Egan (Dublin City University, Ireland)
Markus Grassl (University of Gdansk, Poland)
Assaf Golberger (Tel Aviv University, Israel)
Daniel Gordon (Center for Communications Research, USA)
Dardo Goyeneche (Pontificia Universidad Católica de Chile, Chile)
Jonathan Jedwab (Simon Fraser University, Canada)
Hadi Kharaghani (University of Lethbridge, Canada)
Ilias Kotsireas (Wilfrid Laurier University, Canada)
Máté Matolcsi (Renyi Institute of Mathematics, Hungary)
Koji Momihara (Kumamoto University, Japan)
Padraig Ó Catháin (Dublin City University, Ireland)
Jennifer Seberry (University of Wollongong, Australia)
Sho Suda (National Defense Academy of Japan, Japan)
Ferenc Szöllósi (Shimane University, Japan)
Stefan Weigert (University of York, UK)
Mihály Weiner (Institute of Mathematics, BME, Hungary)
Karol Zyczkowski (Jagiellonian University, Poland)

Scientific Committee:

Víctor Álvarez
Dane Flannery
Kathy Horadam
Hadi Kharaghani
Máté Matolcsi
Jennifer Seberry
Karol Zyczkowski

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- **Dane Flannery**, University of Galway, Ireland.
- **Kathy Horadam**, RMIT University, Australia.
- **Hadi Kharaghani**, Lethbridge University, Canada.
- **Máté Matolcsi**, Renyi Institute of Mathematics, Hungary.
- **Jennifer Seberry**, University of Wollongong, Australia.
- **Karol Zyczkowski**, Jagiellonian University / Center for Theooretical Physics, Poland.

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- **Félix Gudiel**, Universidad de Sevilla, Spain.
- **Belén Güemes**, Universidad de Sevilla, Spain.

Editors: Raúl Falcón and Félix Gudiel.

All the abstracts have been prepared by the authors.

Website: `https://eventos.us.es/go/hadamard2025`

Contents

Welcome

It is our pleasure to welcome you to Seville for **HADAMARD 2025**! Seville is the capital city of Andalusia, multicultural land of painters, writers and artists for hundred of years, and birthplace of flamenco and tapas.

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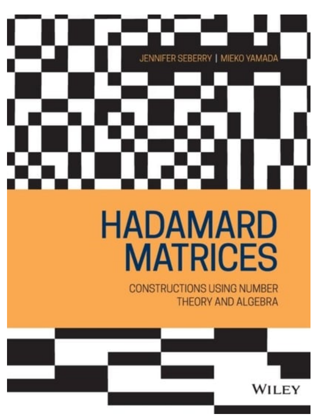
HADAMARD 2025 is the eighth in a series of conferences, the previous seven being held at Wollongong, Australia (1993); Seville, Spain (2007); Galway, Ireland (2009); Melbourne, Australia (2011); Lethbridge, Canada (2014); Budapest, Hungary (2017); Krakow, Poland (2022). The 1993 conference, called the "Hadamard Centenary" in recognition of Hadamard's 1893 paper, was organized by Jennifer Seberry and colleagues.

The purpose of the workshop is to bring together researchers and students interested in design theory, especially as it relates to Hadamard matrices and their applications, as well as in related areas in coding theory, association schemes, sequences, finite geometry, difference sets, quantum information theory, theoretical physics and computer security. The audiences would learn about the latest developments in these areas, discuss the latest findings, take stock of what remains to be done on classical problems and explore different visions for setting the direction for future work.

This edition takes place from May 26 to 30, 2025, at the *Institute of Mathematics of the University of Seville* (IMUS). In HADAMARD 2025, we have 21 invited talks, and 36 contributed talks accepted by the Scientific Committee. In total, 75 participants from 24 different countries around the world.

Both the Organizing and the Scientific Committees thank all the participants for their interest and contribution to make this conference an important scientific event.

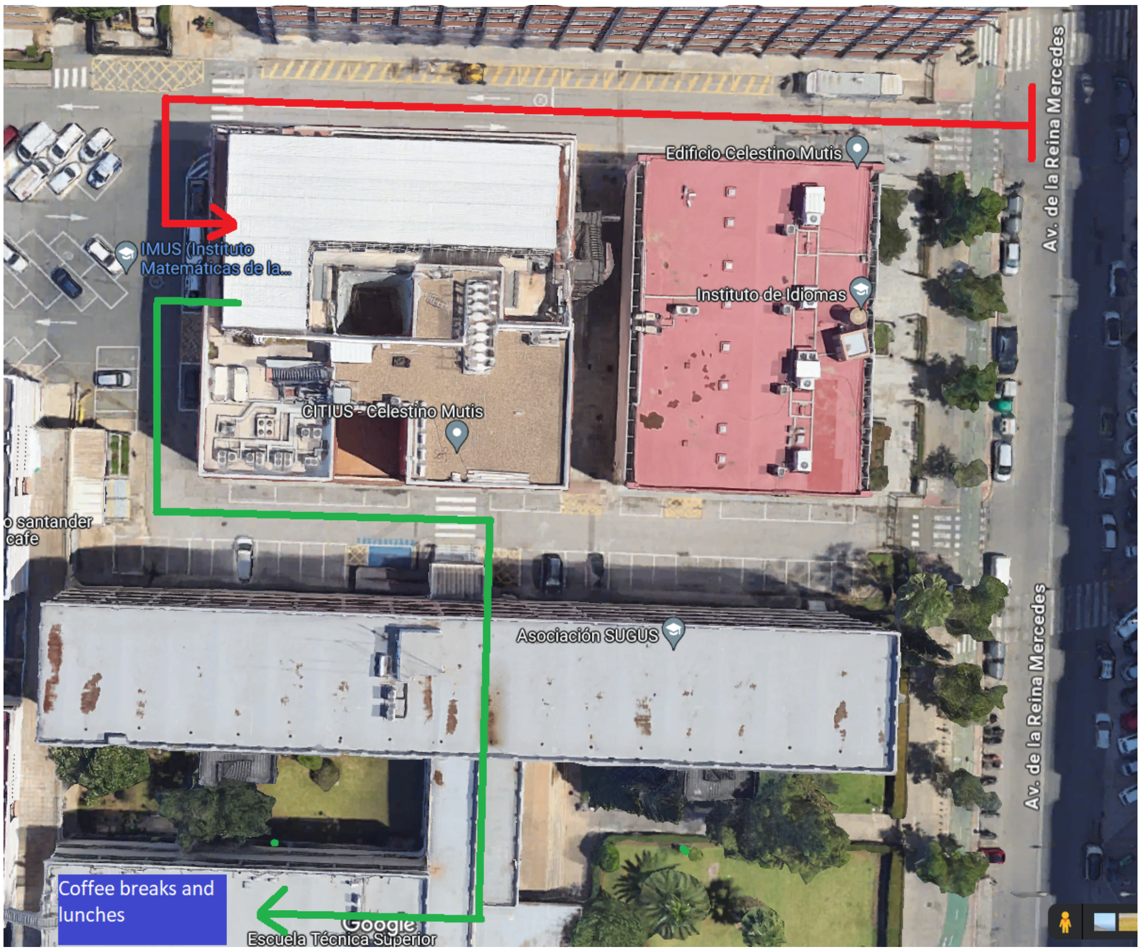
We are specially grateful to Jennifer Seberry, whose generous donation of 15 copies of her book (with Mieko Yamada) will be distributed among the students attending the conference



Venue

The conference takes place at the ***Institute of Mathematics of the University of Seville*** (***IMUS***). It is located on the *Reina Mercedes Campus*, in the southern part of the city. More precisely, the address is:

Edificio Celestino Mutis - CITIUS II, Av. de la Reina Mercedes, s/n, 41012 Sevilla.



The IMUS aims are to organize and develop research activities in all fields and aspects of mathematics and its applications, to promote qualitatively and quantitatively this research, to support the research groups in mathematics of the University of Seville and to foster collaboration between them, with other national and international research groups, in particular by promoting the interdisciplinarity, and with scientific, technological, health, and financial sectors, among others, which could ask for help from mathematics.

Conference programme

Monday, May 26

8:00 - 8:45	REGISTRATION
8:45 - 9:00	OPENING
9:00 - 9:40	[Session chair: Andrés Armario] Hadi Kharaghani *On Hadamard matrices of order* $4n^2$*, n odd*
9:40 - 10:20	Sho Suda *A construction of Hadamard cubes from association schemes on triples*
10:20 - 10:50	COFFEE
10:50 - 11:30	[Session chair: Sho Suda] Koji Momihara *Non-commutative association schemes having divisible design graphs as relations from pseudo-cyclic association schemes*
11:30 - 12:10	Jonathan Jedwab *The maximum number of mutually orthogoval Desarguesian affine planes of order* 2^n
12:10 - 12:50	Daniel M. Gordon *Cyclic Relative Difference Sets*
13:00 - 14:30	LUNCH
14:30 - 14:50	[Session chair: Dean Crnković] Vedran Krčadinac *On higher-dimensional Hadamard matrices and designs*
14:50 - 15:10	Lucija Relić *Projection cubes of symmetric designs*
15:10 - 15:30	Andrea Švob *Switching for 2-designs and applications*
15:30 - 15:50	Travis Dillon *Fixed-strength spherical designs*
15:50 - 16:20	COFFEE
16:20 - 16:40	[Session chair: Félix Gudiel] Daniel Šanko *Legendre pairs, balanced incomplete block designs and codes*
16:40 - 17:00	Oisin Campion *New Constructions of Quantum MDS codes*
18:00 - 20:00	INDIVIDUAL VISIT TO ROYAL ALCÁZAR
20:00 - ...	Restaurante El Cabildo (Tapas)

Tuesday, May 27

9:00 - 10:00	[Session chair: Padraig Ó Catháin] Dane Flannery *Perspectives on algebraic design theory*
10:00 - 10:20	Santiago Barrera Acevedo *Automorphisms of Kimura Hadamard Matrices*
10:20 - 10:50	COFFEE
10:50 - 11:30	[Session chair: Dane Flannery] Padraig Ó Catháin *A new perspective on cocyclic development*
11:30 - 12:10	Ronan Egan *A generalisation of bent vectors for Butson Hadamard matrices*
12:10 - 12:50	Assaf Goldberger *Solving the inverse Gram problem over commutative matrix $*$-algebras over $\mathbb{Z}$ using lattice methods*
13:00 - 14:30	LUNCH
14:30 - 14:50	[Session chair: Assaf Goldberger] Sergey Goryainov *Three results on divisible design graphs*
14:50 - 15:10	Anwita Bhowmik *Divisible design graphs from symplectic graphs over rings with precisely three ideals*
15:10 - 15:30	Gary Greaves *How to design a graph with three eigenvalues*
15:30 - 15:50	Vladislav V. Kabanov *New construction of divisible design graphs using affine designs*
15:50 - 16:20	COFFEE
16:20 - 16:40	[Session chair: Ronan Egan] Tin Zrinski *Constructing directed strongly regular graphs using their orbit matrices and genetic algorithm*
16:40 - 17:00	Patrick Browne *Erdős–Ko–Rado type problems in root systems*
17:00 - 17:20	Struan McCartney *New CEDFs and Related Constructions*
17:20 - 17:40	Shuxing Li *Partial Difference Sets: Broadening the Scope of the Denniston Family*
19:30	*FLAMENCO*

Wednesday, May 28

9:00 - 9:40	[Session chair: Robert Craigen] Jennifer Seberry *Census of symmetric and skew Hadamard matrices of order 4v for odd* $v < 2500$
9:40 - 10:20	Ilias S. Kotsireas *B. O. & T. Q.*
10:20 - 10:50	COFFEE
10:50 - 11:30	[Session chair: Ilias Kotsireas] Dean Crnković: *Self-orthogonal and LCD subspace codes*
11:30 - 12:10	Mihaly Weiner *Intrinsic volumes of the quantum state space and mutually unbiased bases*
12:10 - 12:50	Dardo Goyeneche *Searching for mutually unbiased bases in non-prime power dimensions*
13:00 - 14:30	LUNCH
14:30 - 14:50	[Session chair: Mihaly Weiner] Doris Dumičić Danilović: *On some Hadamard 2-designs and related codes*
14:50 - 15:10	Reza Dastbasteh *Generalized bicycle codes with small connectivity and their properties*
15:10 - 15:30	Emre Güday *Conditional Recurrences and 2-Quasi-Cyclic Codes*
15:30 - 15:50	Tabriz Popatia *Additive Codes and Projective Geometries*
15:50 - 16:20	COFFEE
17:30 - ...	POTTERY (*Barroazul*)

Thursday, May 29

9:00 - 10:00	[Session chair: Hadi Kharaghani] Robert Craigen *Signed groups and Connections to Hadamard (etc) matrices*
10:00 - 10:20	Shalom Eliahou *On 64-modular Hadamard matrices*
10:20 - 10:50	COFFEE
10:50 - 11:30	[Session chair: Máte Matolcsi] Ingemar Bengtsson *Some applications of the discrete Fourier transform to the SIC-POVM problem*
11:30 - 12:10	Karol Życzkowski *Complex Hadamard matrices with special structure*
12:10 - 12:30	Wojciech Bruzda *Isolated Hadamard Matrices*
12:30 - 12:50	Sara D. Cardell *A Study on Binomial Sequences over Finite Fields*
12:50 - 13:00	GROUP PHOTO At the entrance of ETSII
13:00 - 14:30	LUNCH
14:30 - 14:50	[Session chair: Jonathan Jedwab] Petr Lisoněk *On a new class of Hadamard matrices*
14:50 - 15:10	Tuomo Valtonen *Novel non-affine families of 8 × 8 complex Hadamard matrices*
15:10 - 15:30	Pekka Lampio *Classification of weighing matrices*
15:30 - 15:50	Ondřej Turek *Circulant complex Cretan matrices*
15:50 - 16:20	COFFEE
16:20 - 16:40	[Session chair: Raul Falcón] Tekgül Kalaycı *Bent partitions, vectorial dual-bent functions, and LP-Packings*
16:40 - 17:00	Ferruh Özbudak *On Alltop functions, p-ary Alltop functions and almost Hadamard matrices*
17:00 - 17:20	Leonie Scheeren *Complex projective 4-designs as orbits of Clifford-Weil groups*
17:20 - 17:40	Daniel McNulty *Joint measurements from resolvable designs*

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17:40 - 18:00	Rakesh Kumar *Exploring the presence* 4^{th} *MUB in the zero-entanglement subspace of* C^6
18:00 - 18:20	Sergi Sánchez-Aragón *Weight distributions of* $\mathbf{Z}_{p^s}$*-linear simplex and MacDonald codes*
18:20 - 18:40	Paul Leopardi *Boolean-Cayley-graphs: Using Sage and Python software to explore Boolean functions, their Cayley graphs and associated structures*
18:40 - 19:00	Domingo Gómez-Pérez *Recent Advances in the Construction of Hadamard Matrices Using Legendre Pairs*

20:30 - 22:00	CONFERENCE DINNER (*Doña Clara*)

Friday, May 30

	[Session chair: Victor Álvarez]
9:00 - 9:40	Máte Matolcsi *Triplets of mutually unbiased bases*
9:40 - 10:20	Ferenc Szöllősi *Old and new results on Hadamard matrices of order* 6
10:20 - 10:50	COFFEE
	[Session chair: Ferenc Szöllősi]
10:50 - 11:30	Markus Grassl *Exact SIC-POVMs from Permutation Symmetries*
11:30 - 12:10	Stefan Weigert *Mutually Unbiased Bases For Continuous Variables*
12:10 - 12:30	Balázs Pozsgay *Close-to-perfect tensors for holographic error correction codes*
12:30 - 12:50	Víctor Sotomayor *On dualities of dihedral codes, generalised quaternion codes, and associated quantum codes*
12:50 - 13:00	CLOSING
13:00 - 14:30	LUNCH

INVITED TALKS

Some applications of the discrete Fourier transform to the SIC-POVM problem

Thursday
10h50

Ingemar Bengtsson

Stockholms Universitet, Fysikum

Abstract

Several interesting structures in finite dimensional Hilbert space hinge on Heisenberg groups, and Heisenberg groups enjoy symplectic automorphisms. For SIC-POVMs symplectic transformations of order three are of special interest. The discrete Fourier transform is of order 4. I will give a number of examples where the discrete Fourier transform plays an important role in the SIC-POVM problem. Some of them are taken from the literature **[1]**, **[2]**, **[3]**, some of them are new.

References

[1] A. Einstein, Méthode pour la détermination de valeurs statistique d'observations concernant des grandeurs soumises à des fluctuations irrégulièrs. *Archive des Sciences*, **37**: 254-256, 1914.

[2] M. Khaterinejad. On Weyl–Heisenberg orbits of equiangular lines. *Journal of Algebraic Combinatorics*, **28**:333-349, 2008.

[3] M. Appleby, I. Bengtsson, M. Grassl, M. Harrison and G. McConnell. SIC-POVMs from Stark units: Prime dimensions $n^2 + 3$. *Journal of Mathematical Physics*, **63** (11), 2022.

Signed groups and Connections to Hadamard (etc) matrices

Thursday
9h00

Robert Craigen

University of Manitoba

Abstract

Independently—indeed literally poles apart geographically—during the late 1980s, two seemingly very different algebraic ideas, signed groups (R. Craigen) and cocyclic index functions (W. de Launey), crept into the study of Hadamard (etc) matrices and designs. Despite their apparent conceptual-level variance, both ideas arise from largely the same underlying mathematical nuts and bolts.

Such convergence seems to indicate an idea whose time has come . . . even more so if N. Ito's independent introduction, only slightly later, of Hadamard groups—utilizing that same machinery in yet another way—is taken into account. Further, a substantial body of work has resulted . . . from both starting points. Not, as one might suppose, parallel workings of the same theory but at least two quite different bodies of work, diverging both in theoretical impact and in character, apparently illustrating how different formulations of the same machinery may achieve essentially different ends.

Briefly, a **signed group** is a group with a distinguished central element of order 2, denoted multiplicatively as -1. The relationship of signed groups to groups may be regarded as analogous to the relationship of rooted trees to trees.

The impact of cocyclic development of designs, expertly worked out by de Launey, Flannery, Horadam and others is now "well known"; Hadamard groups and signed groups less so. My goal in this talk is to familiarize my audience with the latter, survey the greatest hits (so far) of signed groups and to peel back some more obscure but useful connections.

Self-orthogonal and LCD subspace codes

Wednesday
10h50

Dean Crnković

University of Rijeka - Faculty of Mathematics

(Joint work with Keita Ishizuka, Hadi Kharaghani, Sho Suda and Andrea Švob)

Abstract

In 2008, Kötter and Kschischang introduced subspace codes and propose their applications in error correction for random network coding. Self-orthogonal and LCD subspace codes were introduced recently. In this talk, we will give constructions of self-orthogonal and LCD subspace codes from mutually unbiased and quasi-unbiased weighing matrices, linked systems of symmetric designs and symmetric group divisible designs, and equitable partitions of association schemes and Deza graphs.

A generalisation of bent vectors for Butson Hadamard matrices

Tuesday
11h30

Ronan Egan

Dublin City University, School of Mathematical Sciences

(Joint work with José Andrés Armario, Hadi Kharaghani and Padraig Ó Catháin)

Abstract

An $n \times n$ complex matrix M with entries in the k^{th} roots of unity which satisfies $MM^* = nI_n$, where M^* denotes the conjugate transpose of M, is called a Butson Hadamard matrix. While a matrix with entries in the k^{th} roots typically does not have an eigenvector with entries in the same set, such vectors and their generalisations turn out to have multiple applications. In this talk, an M-bent vector is a column vector $\mathbf{x}$ satisfying $M\mathbf{x} = \sqrt{n}\mathbf{y}$ where $\mathbf{x}$ and $\mathbf{y}$ have entries in the k^{th} roots of unity. In particular we study the special case where $\mathbf{y} = \lambda\overline{\mathbf{x}}$, where $\overline{\mathbf{x}}$ is obtained from $\mathbf{x}$ by replacing the entries with their complex conjugate, and λ is a complex number of modulus 1. Such a vector is called a conjugate self-dual bent vector for M.

We will discuss some techniques from algebraic number theory, used to prove some order conditions and non-existence results for self-dual and conjugate self-dual bent vectors. On the existence side, we give examples of many matrices admitting bent vectors using tensor constructions and Bush-type matrices. We conclude with an application to the covering radius of certain non-linear codes generalising the Reed Muller codes.

Perspectives on algebraic design theory

Tuesday
9h00

DANE FLANNERY

UNIVERSITY OF GALWAY - SCHOOL OF MATHEMATICAL AND STATISTICAL SCIENCES.

Abstract

This is a good time to reflect on some personal experiences in algebraic design theory; in particular to acknowledge fortunate collaborations and to indicate avenues for possible future work.

Solving the inverse Gram problem over commutative matrix $*$-algebras over $\mathbb{Z}$ using lattice methods

Tuesday
12h10

Assaf Goldberger

Tel-Aviv University

Abstract

Let $G \in \mathbb{Z}^{n\times n}$ be a positive-definite matrix. The inverse Gram problem (IGP) over $\mathbb{Z}$ is to find a solution $X \in \mathbb{Z}^{n\times n}$ to the equation $XX^\top = G$. This problem arises naturally in the contexts of Hadamard and weighing matrices, and more generally block designs. In a previous work with Y.Strassler **[1]** we have suggested a lattice based method for solving the IGP. While in some cases our method is effective, it is usually very bad in the contexts mentioned above. In this work we will study the structured version, where X comes from a commutative matrix $*$-algebra $\mathcal{A} \subset \mathbb{Z}^{n\times n}$. Our adapted algorithm becomes effective again, even at instances where the general algorithm fails. So for example, solving the IGP for circulant matrices of size 100 can be done very quickly. In this talk we will also give an upper bound to the number of solutions in $\mathcal{A}$.

References

[1] A. Goldberger and Y. Strassler. A practical algorithm for completing half-Hadamard matrices using LLL. *Journal of Algebraic Combinatorics*, **55**:217–244, 2022.

Cyclic Relative Difference Sets

Monday
12h10

Daniel M. Gordon

IDA Center for Communications Research - La Jolla

Abstract

Let G be a group of order mn with a normal subgroup N of order n. An (m, n, k, λ)-relative difference set (RDS) of G relative to N is a set R of k elements of G such that every element of $G \backslash N$ occurs exactly λ times as a difference of distinct elements of R, i.e. in the group ring $\mathbb{Z}[G]$, R satisfies:

$$RR^{-1} = k + \lambda(G - N).$$

Any relative difference set is a lifting of an $(m, k, n\lambda)$-difference set in G/N. All known nontrivial difference sets with liftings to a cyclic RDS have parameters of complements of classical Singer difference sets:

$$\left(\frac{q^{d+1}-1}{q-1}, q^d, q^{d-1}(q-1)\right).$$

In this talk we give new evidence supporting the conjecture that these are the only ones, and discuss connections to circulant weighing matrices.

Searching for mutually unbiased bases in non-prime power dimensions

Wednesday
12h10

Dardo Goyeneche

Pontificia Universidad Católica de Chile - Faculty of Physics

(Joint work with Fernando Chavez, Gustavo Lara)

Abstract

Mutually unbiased bases (MUBs) play a fundamental role in quantum information theory, yet their full characterisation remains an open challenge **[1]**. Although infinite families of incomplete sets of MUBs exist in non-prime-power dimensions **[2], [3], [4], [5]**, none appear to be extendable to a maximal set of $d+1$ MUBs in dimension d. In this talk, we present two advanced computational frameworks for numerically investigating the existence of structured constellations in finite-dimensional Hilbert spaces. One of these frameworks harnesses GPU acceleration to significantly enhance computational efficiency, enabling the exploration of higher-dimensional spaces with greater accuracy. As a key application, we employ these tools to examine the existence of MUBs in non-prime-power dimensions ranging from 10 to 30, offering new insights into this longstanding open problem.

References

[1] Durt, T., Englert, B. G., Bengtsson, I., Życzkowski, K. On mutually unbiased bases. *International Journal of Quantum Information*, **8**(4):535-640, 2010.

[2] Jaming, P., Matolcsi, M., Móra, P., Szöllősi, F., Weiner, M. A generalised Pauli problem and an infinite family of MUB-triplets in dimension 6. *Journal of Physics A: Mathematical and Theoretical*, **42**(24):245305, 2009.

[3] Goyeneche, D. Mutually unbiased triplets from non-affine families of complex Hadamard matrices in dimension 6. *Journal of Physics A: Mathematical and Theoretical*, **46**(10):105301, 2013.

[4] Goyeneche, D., Gomez, S. Mutually unbiased bases with free parameters. *Physical Review A*, **92**(6):062325, 2015.

[5] M Matolcsi, M., Matszangosz, Á. K., Varga, D., Weiner, M. Triplets of mutually unbiased bases. *arXiv preprint*, arXiv:2503.14752, 2025.

Exact SIC-POVMs from Permutation Symmetries

Friday 10h50

Markus Grassl

University of Gdansk – International Centre for Theory of Quantum Technologies

Abstract

A SIC-POVM corresponds to a set of d^2 complex equiangular lines in dimension d. More than 25 years ago, Gerhard Zauner conjectured that SIC-POVMs exist in all dimensions. Despite significant progress, a proof of their existence for infinitely many dimensions has yet to be found.

Initially, solutions were obtained using numerical optimization or rather complex Gröbner basis calculations [**4**]. There is a general approach that, based on some conjectures, allows to convert numerical solutions into exact ones [**1**]. That approach is based on the corresponding rank-one projection operator with $O(d^2)$ parameters. We show that in certain dimensions, permutation symmetries allow to directly deduce an exact so-called fiducial vector with $O(d)$ parameters. This technique has enabled us to convert numerical solutions from Stark units [**2**] as well as from the approach in [**3**] into exact ones.

M. Grassl's research is carried out under IRA Programme, project no. FENG.02.01-IP.05-0006/23, financed by the FENG programme 2021–2027, Priority FENG.02, Measure FENG.02.01, with the support of the FNP.

References

[1] Marcus Appleby, Tuan-Yow Chien, Steven Flammia, and Shayne Waldron. Constructing exact symmetric informationally complete measurements from numerical solutions. *Journal of Physics A*, **51**:165302, 2018.

[2] Marcus Appleby, Ingemar Bengtsson, Markus Grassl, Michael Harrison, and Gary McConnell. SIC-POVMs from Stark units: Prime dimensions $n^2 + 3$, *Journal of Mathematical Physics*, **63**:112205, 2022.

[3] Marcus Appleby, Steven T. Flammia, and Gene S. Kopp. A Constructive Approach to Zauner's Conjecture via the Stark Conjectures. Preprint arXiv:2501.03970 [math.NT], 2025.

[4] Andrew J. Scott and Markus Grassl. Symmetric informationally complete positive-operator-valued measures: A new computer study. *Journal of Mathematical Physics*, **51**:042203, 2010.

The maximum number of mutually orthogoval Desarguesian affine planes of order 2^n

Monday
11h30

Jonathan Jedwab

Simon Fraser University - Mathematics

(Joint work with James A. Davis, Shuxing Li, Jingzhou Na, Thomas Pender, Tabriz Popatia)

Abstract

Two affine planes of the same order and on the same pointset are *orthogoval* if each line of one plane intersects each line of the other plane in at most two points [**1**]. A set of pairwise orthogoval affine planes is *mutually orthogoval*.

I shall describe a new computational method to determine the maximum number of Desarguesian mutually orthogoval affine planes of order 2^n. This gives exact results for orders 8 and 16, and indicative results for orders 32 and 64.

References

[1] C. J. Colbourn, C. Ingalls, J. Jedwab, M. Saaltink, K. W. Smith, B. Stevens. Sets of mutually orthogoval projective and affine planes. *Combinatorial Theory*, **4**:#8, 2024.

On Hadamard matrices of order $4n^2$, n odd

Monday
9h00

Hadi Kharaghani

University of Lethbridge - Department of Mathematics and Computer Science /Lethbridge, Alberta, Canada

(Joint work with Darcy Best, Behruz Tayfeh-Rezaie, and Vlad Zaitsev)

Abstract

A new class of Hadamard matrices will be introduced, with a particular emphasis on those of order 36

B. O. & T. Q.

Wednesday
9h40

Ilias S. Kotsireas

University - CARGO Lab, Wilfrid Laurier University, Waterloo, ON, Canada

Abstract

We shall explain the meaning of the cryptic title "B. O. & T. Q.", an acronym inspired by one of the birthday boys! Subsequently, we will describe several such new B. O. & T. Q. pertaining to:

[1] Periodic Golay pairs

[2] Legendre pairs

[3] quaternary Legendre pairs

[4] Hadamard matrices

[5] weighing matrices

[6] T-sequences

[7] Base sequences

[8] Orthogonal Designs

[9] Turyn-type sequences

As an important corollary, we provide several different proofs of Marco Buratti's motivation theorem, presented at CODESCO 2024 in Sevilla.

Triplets of mutually unbiased bases

Friday
9h00

Máte Matolcsi

Alfréd Rényi Institute of Mathematics

(Joint work Ákos Matszangosz, Dániel Varga, Mihály Weiner)

Abstract

The concept of mutually unbiased bases (MUBs) is fundamental in quantum information theory. It is well-known that pairs of MUBs correspond to complex Hadamard matrices. In this talk, I describe a systematic study of triplets of MUBs, and report some recent progress [**1**]. Namely, we identify the fundamental algebraic relations ("cube axioms") defining MUB-triplets in any dimension d, and provide a conjectured identity, which would prove the non-existence of quadruplets of MUBs in dimension 6, using a recent result of [**2**]. Numerical evidence heavily supports the validity of the conjecture, and a formal proof may be achieved with the help of the cube axioms.

References

[1] M. Matolcsi, Á. Matszangosz, D. Varga, M. Weiner. Triplets of mutually unbiased bases. *preprint on arxiv*, 2025.

[2] Á. Matszangosz, F. Szöllősi. A characterization of complex Hadamard matrices appearing in families of MUB triplets. *Des. Codes Cryptogr.*, **92**:4313-4333, (2024).

Non-commutative association schemes having divisible design graphs as relations from pseudo-cyclic association schemes

Monday
10h50

Koji Momihara

Kumamoto University, Division of Natural Science, Faculty of Advanced Science and Technology

(Joint work with Sho Suda)

Abstract

It is known that by replacing the entry $+1$ by I_2 and the entry -1 by $J_2 - I_2$ in a Hadamard matrix, the resulting matrix forms an incidence matrix of a symmetric group divisible design. Gibbons-Mathon **[1]** generalized this construction of symmetric group divisible designs based on balanced generalized weighing matrices.

In this talk, we modify Gibbons-Mathon's construction by using d-class pseudo-cyclic symmetric association schemes to obtain $(2d-1)$-class non-commutative association schemes such that exactly d nontrivial relations are divisible design graphs. Furthermore, we discuss the problem on isomorphism between non-commutative association schemes obtained by our construction, which is related to a problem on normalization of balanced generalized weighing matrices. In particular, when $d = 2$, the problem is described in terms of Godsil-McKay switching for conference graphs. For example, we claim the following. Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be two non-isomorphic 2-class pseudo-cyclic symmetric association schemes. If any association scheme obtained by applying Godsil-McKay switching to the conference graphs in $\mathcal{A}_1$ is not isomorphic to $\mathcal{A}_2$, then the non-commutative association schemes obtained by applying our construction to $\mathcal{A}_1$ and $\mathcal{A}_2$ are non-isomorphic.

References

[1] P. B. Gibbons, R. Mathon, Construction methods for Bhaskar Rao and related designs, *J. Austral. Math. Soc. Ser. A* **42** 5–30, (1987).

A new perspective on cocyclic development

Tuesday
10h50

Padraig Ó Catháin

Dublin City University - Fiontar agus Scoil na Gaeilge

(Joint work with Santiago Barrera Acevedo, Heiko Dietrich and Ronan Egan)

Abstract

The theory of cocyclic development was developed by de Launey, Flannery and Horadam in the 1990s, and found a spiritual home in Seville in the 2000s. In this talk, I will report on recent work with Santiago Barrera Acevedo, Heiko Dietrich and Ronan Egan on the centraliser algebras of monomial representations. Group development and cocyclic development occur as special cases of this framework, corresponding to regular permutation groups and their central extensions. Our framework allows us to work with arbitrary monomial groups however.

One of the bottlenecks in the construction of cocyclic Hadamard matrices is that the space of 2-cocycles grows exponentially with the size of the indexing group. In contrast, I will show that searching for real or complex Hadamard matrices in the centraliser algebra of a monomial representation of bounded rank is tractable (if not always very practical).

This work is dedicated to Prof Dane Flannery, on the occasion of his 60th birthday.

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[1] Santiago Barrera Acevedo, Padraig Ó Catháin, Heiko Dietrich and Ronan Egan. Centralisers of monomial representations and applications in combinatorics. *Algebraic Combinatorics, to appear.*

Census of symmetric and skew Hadamard matrices of order $4v$ for odd $v < 2500$

Tuesday
9h00

Dragomir Z. Đoković and Jennifer Seberry

University of Waterloo University of Wollongong

(Joint work with Amin Bahmanian)

Abstract

An up-to-date listing of the existence of symmetric and skew Hadamard matrices of order $4v$ for odd $v < 2500$ is given. In addition an existence theorem for asymptotic skew Hadamard matrices is given for prime powers $\equiv 3 \mod 4$ and those $\equiv 5 \mod 8$.

A construction of Hadamard cubes from association schemes on triples

Monday
9h40

Sho Suda

National Defense Academy of Japan, Department of Mathematics

(Joint work with Amin Bahmanian)

Abstract

A d-dimensional Hadamard matrix of order n is a d-dimensional matrix H of order n with entries $1, -1$ such that for any j and any a, b,

$$\sum_{1 \leq x_1, \ldots, \hat{x_j}, \ldots, x_d \leq n} H(x_1, \ldots, a, \ldots, x_d) H(x_1, \ldots, b, \ldots, x_d) = n^{d-1} \delta_{ab}.$$

A three-dimensional Hadamard matrix is said to be a Hadamard cube. In **[2]**, Krčadinac, Pavčević, and Tabak used finite fields to construct Hadamard cubes of order $p+1$ where p is a prime power.

Constructions of 2-dimensional Hadamard matrix vary widely. Goethals and Seidel **[1]** in 1970 showed that a regular symmetric Hadamard matrix with constant diagonal entries is constructed as a linear combination of the identity matrix, and adjacency matrices of a strongly regular graph with certain parameters and its complement. Note that these three $(0, 1)$-matrices define an association scheme.

In this talk, we provide a construction of Hadamard cubes as a linear combination of adjacency matrices of association schemes on triples with certain parameters **[3]**. Moreover, we will give a construction of association schemes on triples with the desired parameters from any conference matrix. In particular, we prove the following: Let n be the order of a conference matrix. Then there exists a Hadamard cube of order n. .

References

[1] J. M. Goethals, J. J. Seidel, Strongly regular graphs derived from combinatorial designs. *Canad. J. Math.* **22**: 597–614, (1970).

[2] V. Krčadinac, M. O. Pavčević, K. Tabak, Three-dimensional Hadamard matrices of Paley type. *Finite Fields and Their Applications*, **92**: 102306, (2024).

[3] D.M. Mesner, P. Bhattacharya, Association schemes on triples and a ternary algebra, *J. Combin. Theory Ser. A*, **55**: 204–234, (1990).

Old and new results on Hadamard matrices of order 6

Friday
9h40

Ferenc Szöllősi

Shimane University, Department of Mathematical Sciences

(Joint work in part with Á. K. Matszangosz)

Abstract

I will review some constructions of Hadamard matrices **[2]** of order 6, and give a characterization of interesting subfamilies **[1]**.

References

[1] Á. K. Matszangosz, F. Szöllősi: A characterization of complex Hadamard matrices appearing in families of MUB triplets, Des. Codes Cryptogr., 92, 4313–4333 (2024).

[2] F. Szöllősi: Complex Hadamard matrices of order 6: A four-parameter family, J. Lond. Math. Soc. (2), 85:3, 616–632 (2012).

Mutually Unbiased Bases For Continuous Variables

Friday 11h30

Stefan Weigert

University of York, Department of Mathematics, York
United Kingdom

(Joint work with Adam Beales)

Abstract

Mutually unbiased bases **[1], [2]** emerge naturally in infinite-dimensional Hilbert spaces **[3]** when discussing quantum systems with position and momentum observables, known as (pairs of) continuous variables. We describe similarities and differences between mutually unbiased bases—and, hence Hadamard matrices—arising in finite- and infinite-dimensional Hilbert spaces, respectively. The maximal number of mutually unbiased bases for continuous variables (with basis-independent overlaps) is not known even in the simplest case of a single pair of canonically conjugate observables, corresponding to $N = 1$. For this case, we point out a link between the known triple of mutually unbiased bases and the number of equiangular lines in $\mathbb{R}^2$, the "phase space" of the system. We conjecture that this relation extends to more pairs of continuous variables, i.e. to $N > 1$: one can construct at least as many mutually unbiases bases as there are equiangular lines in $\mathbb{R}^{2N}$.

References

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[2] D McNulty and S Weigert. Mutually Unbiased Bases in Composite Dimensions – A Review. arXiv:2410.23997 [quant-ph].

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Intrinsic volumes of the quantum state space and mutually unbiased bases

Wednesday
11h30

MIHALY WEINER

BUDAPEST UNIVERSITY OF TECHNOLOGY AND ECONOMICS

(Joint work with Zs. Szilágyi)

Abstract

Previous studies about the geometrical properties of the state space of a d-level quantum system have determined its volume and surface area; see [**1**]. Building on this foundation, we derive explicit formulas [**2**] for two additional intrinsic volume quantities.

The question of whether a complete set of mutually unbiased bases exists in dimension d can be equivalently framed as whether a specific convex polytope can be inscribed within the state space of a d-level quantum system [**3**]. One motivation for our work was the hypothesis that a smaller intrinsic volume of the state space compared to the corresponding intrinsic volume of the mentioned polytope could rule out such an inscription.

While our computations of these two intrinsic volumes do not lead to this conclusion, they nonetheless provide fundamental insights into the geometric structure of quantum state spaces. In particular, we show that these quantities can be used to rule out the existence of some unit-vector "configuration" (though not the one formed by the bases vectors of a complete set of mutually unbiased bases).

References

[1] K. Zyczkowski and H.-J. Sommers: Hilbert–Schmidt volume of the set of mixed quantum states. *Journal of Physics A: Mathematical and General*, **36**:10115–10130, 2003.

[2] Zs. Szilágyi and M. Weiner: Intrinsic volumes of the quantum state space and mutually unbiased bases. *In preparation*, 2025.

[3] I. Bengtsson and Å. Ericsson: Mutually Unbiased Bases and the Complementarity Polytope. *Open Systems & Information Dynamics*, **12**:107–120, 2005.

Complex Hadamard matrices with special structure

Thursday
11h30

Karol Życzkowski

Jagiellonian University, Cracow & Center for Theoretical Physics, Polish Academy of Sciences, Warsaw

(Joint work with Wojciech Bruzda and Grzegorz Rajchel-Mieldzioć)

Abstract

Recent results on two-unitary complex Hadamard matrices will be presented. Such Hadamard matrices, defined for squared dimensions d^2, remain unitary after partial transpose and reshuffling. These matrices, important for applications in theory of quantum information (generation of absolutely maximally entangled states of four systems with d levels each, quantum error correction codes) do not exist for $d = 2$ but can be constructed **[1]** for $d = 3, 4, 5$. In this talk we are going to present such a matrix for $d = 6$, which belongs to the Butson class $B(36, 6)$ and is directly related to the quantum analog of the Euler problem of 36 officers **[2]**.

References

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CONTRIBUTED TALKS

Automorphisms of Kimura Hadamard Matrices

Tuesday
10h00

Santiago Barrera Acevedo

La Trobe University - Department of Mathematical and Physical Sciences

(Joint work with Melissa Lee)

Abstract

During the classification of Hadamard matrices (HMs) of order 28, Kimura [**1**] reported a class of such matrices with a notable structure derived from a dihedral group. This led Kimura to study HMs with this structure, now known as Kimura Hadamard Matrices (KHMs). Kimura and Niwasaki [**2**], [**3**] presented examples of KHMs for all orders of the form $8k + 4$ where k is an odd integer satisfying $k \leq 41$, except $k = 15$. Following this, Shinoda and Yamada [**4**] discovered a construction of KHMs of orders of the form $8p + 4$ where p is an odd prime satisfying $p \equiv 1 \pmod 4$ and $q = 2p - 1$ is a prime power.

Motivated by a conjecture proposed by Ó Catháin in Section 7.4.1 [**5**], which claims that the order of the automorphism group of a KHM is 2^5k, we computed the automorphism groups of the matrices found by Kimura and Niwasaki, as well as the matrices introduced by Shinoda and Yamada for $p \leqslant 601$. Our computations disprove this conjecture and highlight common structural properties of the automorphism groups of these matrices. For example, they all contain copies of the groups C_2, Q_8, and D_{2k}. Additionally, their orders are of the form $2^t \cdot k$ for $4 \leq t \leq 7$. In some cases, these groups contain a copy of C_4 (distinct from those contained in Q_8) or C_8.

In this talk, we will discuss KHMs, examining their existence and the structure of their automorphism groups as part of ongoing research.

References

[1] H. Kimura. Classification of Hadamard matrices of order 28. *Discrete Mathematics*, **133**: 171–180, 1994.

[2] H. Kimura. Hadamard matrices and dihedral groups. *Designs, Codes and Cryptography*, **9**, 71–77, 1996.

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[4] K. Shinoda and M. Yamada. A family of Hadamard matrices of dihedral type. *Discrete Applied Mathematics*: 141–150, 2000.

[5] P. Ó Catháin. Group actions on Hadamard matrices. Master Thesis, National University of Ireland, Galway: 2008.

A Study on Binomial Sequences over Finite Fields

Thursday
12h30

Miguel Beltrá[1] and Sara D. Cardell[2] and Verónica Requena[1]

[1] Universidad de Alicante, Spain

[2] Universidade Estadual Paulista (Unesp), Brazil

Abstract

The binary binomial sequences correspond to the diagonals of the binary Pascal's triangle. They have interesting properties such as all binary sequences with period a power of 2 can be computed as the XOR of a finite set of binomial sequences [**1**]. Other properties of these sequences (period, linear complexity, construction rules or relations among them) have been deeply analyzed for the binary case [**1**]. In this work, we study the binomial $p-$ary sequences for a prime p, which are the diagonals of the Pascal's triangle modulo p.

Let p be a prime number and $\mathbb{F}_p$ be the Galois field of p elements. We say $\{a_n\}_{n\geq 0} = \{a_0, a_1, a_2, \ldots\}$ is a $p-$**ary sequence** if its terms $a_n \in \mathbb{F}_p$, for $n \geq 0$. The sequence $\{a_n\}_{n\geq 0}$ is **periodic** if and only if there exists an integer T such that $a_{n+T} = a_n$, for all $n \geq 0$. In the sequel, all the sequences considered will be $p-$ary sequences and the term $+$ will denote the sum modulo p.

A binomial sequence $\left\{\binom{n}{k}\right\}_{n\geq 0}$, with $k \geq 0$ being an integer, is a $p-$ary sequence given by

$$\left\{\binom{n}{k}\right\}_{n\geq 0} = \left\{\binom{0}{k}, \binom{1}{k}, \binom{2}{k}, \ldots\right\}_{\bmod p},$$

whose terms are the binomial coefficients $\binom{n}{k}$ reduced modulo p. This sequence is named as the $p-$**ary** $k-$**th binomial sequence** or, simply, $p-$**ary binomial sequence**. Notice that $\binom{n}{k} = 0$ if $k > n$.

Any p-ary sequence $\{s_i\} = \{s_0, s_1, s_2, \ldots, s_{p^L-1}, \ldots\}$ of period $T = p^L$, with L a positive integer, can be written as a linear combination of p-ary binomial sequences. That is, there exist coefficients $c_i \in \mathbb{F}_p$, with $i \in \{0, 1, \ldots, p^L - 1\}$, such that

$$\{s_i\} = c_0\left\{\binom{n}{0}\right\}_{n\geq 0} + c_1\left\{\binom{n}{1}\right\}_{n\geq 0} + c_2\left\{\binom{n}{2}\right\}_{n\geq 0} + \cdots + c_{p^L-1}\left\{\binom{n}{p^L-1}\right\}_{n\geq 0}.$$

This expression is called the **binomial representation** of the sequence $\{s_i\}$.

There exists a relation between a $p-$ary sequence of period p^L and its binomial representation given by

$$\begin{bmatrix} s_0 & s_1 & s_2 & \cdots & s_{p^L-1} \end{bmatrix} = \begin{bmatrix} c_0 & c_1 & c_2 & \cdots & c_{p^L-1} \end{bmatrix} H_L^{(p)}$$

where

$$H_L^{(p)} = \begin{bmatrix} \binom{0}{0} & \binom{1}{0} & \binom{2}{0} & \cdots & \binom{p^L-1}{0} \\ \binom{0}{1} & \binom{1}{1} & \binom{2}{1} & \cdots & \binom{p^L-1}{1} \\ \binom{0}{2} & \binom{1}{2} & \binom{2}{2} & \cdots & \binom{p^L-1}{2} \\ \vdots & \vdots & \vdots & & \vdots \\ \binom{0}{p^L-1} & \binom{1}{p^L-1} & \binom{2}{p^L-1} & \cdots & \binom{p^L-1}{p^L-1} \end{bmatrix} \bmod p,$$

is called $p-$**Hadamard matrix** (or **Pascal matrix**). This matrix satisfies that $H_L^{(p)} = H_1^{(p)} \otimes H_{L-1}^{(p)}$. Moreover, as the inverse of $H_L^{(p)}$ coincides with its transpose with respect to the antidiagonal, we can obtain the coefficients of the binomial representation of $\{s_i\}$, as

$$\begin{bmatrix} c_0 & c_1 & c_2 & \cdots & c_{p^L-1} \end{bmatrix} = \begin{bmatrix} s_0 & s_1 & s_2 & \cdots & s_{p^L-1} \end{bmatrix} \left(H_L^{(p)}\right)^{-1}$$

with

$$\left(H_L^{(p)}\right)^{-1} = \begin{bmatrix} \binom{p^L-1}{p^L-1} & \binom{p^L-1}{p^L-2} & \cdots & \binom{p^L-1}{1} & \binom{p^L-1}{0} \\ \binom{p^L-2}{p^L-1} & \binom{p^L-2}{p^L-2} & \cdots & \binom{p^L-2}{1} & \binom{p^L-2}{0} \\ \vdots & \vdots & & \vdots & \vdots \\ \binom{1}{p^L-1} & \binom{1}{p^L-2} & \cdots & \binom{1}{1} & \binom{1}{0} \\ \binom{0}{p^L-1} & \binom{0}{p^L-2} & \cdots & \binom{0}{1} & \binom{0}{0} \end{bmatrix}.$$

Acknowledgments

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References

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Divisible design graphs from symplectic graphs over rings with precisely three ideals

Tuesday 14h50

Anwita Bhowmik

Hebei Normal University - Postdoctoral Research Station of Mathematics, School of Mathematical Sciences

(Joint work with Sergey Goryainov)

Abstract

A *group divisible design (GDD)* is an incidence structure that consists of v points and b blocks with constant block size k; each point appears in r blocks, and the set of points can be partitioned into m groups each of size n, such that two points from a group occur together precisely in λ_1 blocks and two points from different groups occur together precisely in λ_2 blocks. Bose and Connor introduced GDDs in [**3**]. In [**2**], Bose considered symmetric GDDs with the dual property, which is a special class of GDDs. A graph can be interpreted as an incidence structure by taking the vertices of the graph as points, and the neighbourhoods of the vertices as blocks. A *divisible design graph (DDG)* is a graph whose adjacency matrix is the incidence matrix of a group divisible design. DDGs were introduced in [**7**] and [**4**]. The design corresponding to a DDG is a symmetric GDD with the dual property. In [**5**], Deza graphs were constructed from symplectic graphs over $\mathbb{Z}/p^n\mathbb{Z}$ for a prime p and a positive integer n, and this work was generalized to finite local rings in [**6**]. In [**1**], we slightly modify the definition of a symplectic graph over a ring and consider such graphs over rings with precisely three ideals. This gives two new infinite families of DDGs.

References

[1] A. Bhowmik and S. Goryainov. Divisible design graphs from symplectic graphs over rings with precisely three ideals. *arXiv preprint arXiv:2412.04962*, 2024.

[2] R.C. Bose. Symmetric group divisible designs with the dual property. *Journal of Statistical Planning and Inference*, **Volume 1, Issue 1**: p. 87–101, 1977.

[3] R. C. Bose and W. S. Connor. Combinatorial properties of group divisible incomplete block designs. *The Annals of Mathematical Statistics*, p. 367–383, 1952.

[4] W. H. Haemers, H. Kharaghani and M. A. Meulenberg. Divisible design graphs. *Journal of Combinatorial Theory Series A*, **Volume 118, Issue 3**: p. 978–992, 2011.

[5] Y. Meemark and T. Prinyasart. On symplectic graphs modulo p^n. *Discrete Mathematics*, **Volume 311, Issue 17**: p. 1874–1878, 2011.

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Erdős–Ko–Rado type problems in root systems

Tuesday
16h40

PATRICK BROWNE

UNIVERSITY - TUS IRELAND

(Joint work with P. Ó Catháin & Q. R. Gashi)

Abstract

Given a Lie algebra, two roots are said to be strongly orthogonal if neither their sum nor difference is a root. In this talk, we investigate sets of mutually strongly orthogonal roots. In particular, those such that any two such sets have the property that the difference between their sums can itself be expressed as the sum of a strongly orthogonal set of roots. We discuss this property and its relationship to Erdős–Ko–Rado type problems, with a particular focus on recent results for exceptional type root systems. Lastly we discuss intersections with binary codes and our work.

Isolated Hadamard Matrices

Thursday
12h10

Wojciech Bruzda

Center for Theoretical Physics, Polish Academy of Sciences, Aleja Lotników 32/46, 02-668 Warsaw, Poland

Abstract

We present a method for obtaining a (potentially) infinite sequence of isolated complex Hadamard matrices (CHMs) in dimensions $N > 7$, characterized by special structures in their cores. In particular, this construction yields new representatives of orders 9, 10, and 11, whose elements are not roots of unity, as well as several new multiparametric families of order 10. These results reveal novel connections between certain eight-dimensional matrices and provide new insights into the classification of CHMs of higher orders.

New Constructions of Quantum MDS codes

Monday
16h40

Oisin Campion

University College Dublin - School of Mathematics and Statistics

(Joint work with Fernando Hernando and Gary McGuire)

Abstract

The topic of quantum error-correction has received much attention in the literature in recent years, with many authors focusing on constructing codes that achieve the Quantum Singleton bound. Such codes are called MDS codes, and while many are already known, there are several gaps in the literature. In this talk, we give a brief introduction to quantum stabilizer codes and present some new constructions of previously unknown MDS quantum stabilizer codes. These quantum codes are constructed from classical generalized Reed-Solomon codes that are self-orthogonal with respect to a Hermitian inner-product.

References

[1] B. Barbero-Lucas, F. Hernando, H. Martin-Cruz, G. McGuire. MDS, Hermitian Almost MDS, and Gilbert-Varshamov Quantum Codes from Generalized Monomial-Cartesian Codes. *Quantum Inf. Proess*, 23(86), 2024.

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Generalized bicycle codes with small connectivity and their properties

Wednesday 14h50

Reza Dastbasteh

Department of Basic Sciences, Tecnun - University of Navarra, San Sebastian, Spain

(Joint work with Olatz Sanz Larrarte[1], Josu Etxezarreta Martinez[1], Arun John Moncy[2], Pedro M Crespo[1], and Rubén M Otxoa [3])

Abstract

Generalized bicycle (GB) codes form a family of quantum codes constructed from circulant matrices and are well known for their simple structure and redundancy in low weight stabilizers [**1**]. In this work, we study all GB codes with small connectivity by investigating the structure of their circulant matrices and the quantum code parameters. In particular, we establish the existence of two infinite families of GB codes with connectivity four and parameters $[[d^2+1, 2, d]]$ for an odd d, and $[[d^2, 2, d]]$ when d is even.

We provide a circuit extraction procedure for these codes, develop a syndrome extraction schedule, and investigate their effective minimum distance. Notably, in many cases, the effective minimum distance matches the theoretical distance. We also provide evidence that these codes are suitable for various quantum architectures, such as superconducting qubits and spin qubits.

We characterize the girth of all GB codes and prove that the mentioned families achieve a girth of 8—the highest possible for a GB code. This distinguishes GB codes that may exhibit superior performance in message-passing decoders such as belief propagation (BP) and belief propagation and ordered-statistics decoding (BP+OSD). We are currently analyzing the decoding performance of various GB codes under the specified decoding algorithms.

References

[1] Panteleev P, Kalachev G. Degenerate quantum LDPC codes with good finite length performance. *Quantum*, **5**:585, 2021.

[1]Department of Basic Sciences, Tecnun - University of Navarra, San Sebastian, Spain
[2]Donostia International Physics Center, San Sebastian, Spain.
[3]Hitachi Cambridge Laboratory, Cambridge, United Kingdom.

On some Hadamard 2-designs and related codes

Wednesday
14h30

Doris Dumičić Danilović

University of Rijeka - Faculty of Mathematics

(Joint work with Andrea Švob

Abstract

In this talk, we show results of the construction of Hadamard designs with certain parameters. We classified Hadamard 2-$(51, 25, 12)$ designs having a non-abelian automorphism group of order 10, i.e. $\mathrm{Frob}_{10} \cong Z_5 : Z_2$, where a subgroup isomorphic to Z_2 fixes setwise all the Z_5-orbits of the sets of points and blocks. Further, we show results of the classification of Hadamard 2-$(59, 29, 14)$ designs having a non-abelian automorphism group of order 14, i.e. $\mathrm{Frob}_{14} \cong Z_7 : Z_2$, where a subgroup isomorphic to Z_2 fixes setwise all the Z_7-orbits of the sets of points and blocks. From the obtained Hadamard designs and the corresponding Hadamard matrices, we construct linear self-dual codes.

Fixed-strength spherical designs

Monday
15h30

Travis Dillon

Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA 02141, USA

(Joint work with John Polhill, Ken Smith, and Eric Swartz)

Abstract

A spherical t-design is a finite subset X of the unit sphere such that every polynomial of degree at most t has the same average over X as it does over the entire sphere. Spherical designs were introduced in 1977 by Delsarte, Goethals, and Seidel [**2**], and determining the minimum possible size of spherical designs has been an important research topic since. A main focus of this research has been the search for asymptotic bounds in a fixed dimension as $t \to \infty$, which culminated in 2013 with Bondarenko, Radchenko, and Viazovska's result that minimal t-designs on S^d have $O_d(t^d)$ points [**1**].

The complementary asymptotic regime, where t is fixed and the dimension tends to infinity, is much less studied. Utilizing techniques from a variety of fields, including algebra, geometry, probability and linear programming, we prove a variety of new results in this regime. By establishing an explicit connection with designs over Gaussian space, we show that the size of spherical designs is nearly monotone: If there is a weighted t-design on S^d with N points, then for every $k \leq d$ there is a weighted t-design on S^k with at most $2(t+1)N$ points. We also improve the upper bounds on the size of minimal designs in this regime and establish an optimal asymptotic bound for signed spherical designs.

References

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On 64-modular Hadamard matrices

Thursday
10h00

SHALOM ELIAHOU

UNIVERSITÉ DU LITTORAL CÔTE D'OPALE, CALAIS, FRANCE

Abstract

Let $m \geq 2$ be an integer. Introduced by Marrero and Butson in the early 1970's **[3,4]**, m-modular Hadamard matrices are square matrices H of order n with entries in $\{\pm 1\}$ such that $HH^T \equiv nI_n \bmod m$. Every true Hadamard matrix is m-modular. As a modest partial converse, every m-modular Hadamard matrix of order $n < m$ is a true Hadamard matrix.

As in the classical case, the m-modular Hadamard conjecture states that for every $n \equiv 0 \bmod 4$, there exists an m-modular Hadamard matrix of order n. While this version, for a given modulus m, is weaker than its classical counterpart, Hadamard's conjecture is *equivalent* to the conjunction of its m_i-modular versions for any infinite set of moduli m_i, for instance $m_i = 2^i$. This provides a strong incentive to tackle the m-modular Hadamard conjecture for as large moduli m as possible. The current record is $m = 32$, achieved more than 20 years ago in **[1]**, with an updated solution in **[2]**.

Can one go further and settle the m-modular Hadamard conjecture for $m = 64$? In this talk, we will discuss a computer-aided partial solution of it, namely for all orders $n = 4k$ such that $k \equiv 1, 3, 5 \bmod 16$. These solutions arise, via the Goethals-Seidel array, from a computer-aided construction of 64-modular Golay-Turyn quadruples of length k. One objective in this talk is to motivate people to tackle the five remaining open cases for $m = 64$, namely those orders $n = 4k$ with $k \equiv 7, 9, 11, 13, 15 \bmod 16$.

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Recent Advances in the Construction of Hadamard Matrices Using Legendre Pairs

Thursday 18h40

DOMINGO GÓMEZ-PÉREZ

UNIVERSIDAD DE CANTABRIA - DEPARTAMENTO DE MATEMÁTICAS, ESTADÍSTICA Y COMPUTACIÓN

(Joint work with Ilias Kotsireas and Ana I. Gómez)

Abstract

Legendre pairs are pairs of binary sequences of equal length with the additional property that the sum of their periodic correlation functions is constant, equal to two. These pairs of sequences have attracted interest because they are building blocks for Hadamard matrices, and there is evidence of the existence of such pairs for many lengths. Kotsireas and several authors **[1], [2]** have advanced the understanding of the properties of Legendre pairs for certain lengths, leading to the discovery of new pairs. The core problem of such algorithms is their computational complexity and how to implement them as efficiently as possible. In this talk, we review previous results in this problem, both theoretical and practical. Then, we present an alternative branch-and-bound algorithm and combine it with decimations to speed up the computations. We conclude with some open problems and future work.

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[2] Turner, Jonathan S., et al. "A Legendre pair of length 77 using complementary binary matrices with fixed marginals." *Designs, Codes and Cryptography*, **89**:1321-1333, 2021.

Three results on divisible design graphs

Tuesday
14h30

Sergey Goryainov

Hebei Normal University - School of Mathematical Sciences

(Based on joint work with Elena Konstantinova & Honghai Li, joint work with Anwita Bhowmik & Bart De Bruyn and joint work with Bart De Bruyn, Ruilin Ma & Ruihan Xie)

Abstract

Divisible design graphs were introduced in **[4]** as a bridge between graph theory and design theory. Since then, tens of infinite families of divisible design graphs were introduced.

In Theorem 4.4 **[2]** a construction of thin divisible design graphs was proposed. This construction requires the existence of two agreed square matrices of the same size: a symmetric weighing matrix W and a $\{0, 1, 2\}$-matrix R with the property $R^2 = \alpha I + \beta J$ for some real numbers α and β. In this talk, we discuss a partial result how the latter condition on R can be satisfied with using regular symmetric Hadamard matrices with constant diagonal and formulate a problem of the existence of symmetric weighing matrices whose underlying graph is a complete multipartite graph with parts of the same size.

In **[3]**, a construction of Deza graphs based on symplectic spaces was proposed. In this talk, we present our recent result that these Deza graphs are, moreover, divisible design graphs, and discuss possible generalisation of this construction.

It is well known that hyperbolic and elliptic affine polar graphs (Section 3.3.1 **[1]**), as well as their variations (Section 3.3.2 **[1]**), are strongly regular. It is also known that the parabolic affine polar graphs are not strongly regular. In this talk, we present three new infinite families of divisible design graphs obtained as variations of parabolic affine polar graphs. In particular, we discuss a connection with regular symmetric Hadamard matrices with constant diagonal obtained from the Desarguesian affine planes and present a construction that produces infinitely many divisible design graphs for each Mersenne prime.

References

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[3] J. Guo, K. Wang and F. Li. Deza graphs based on symplectic spaces. *European Journal of Combinatorics*, **Volume 31, Issue 8**:pages 1969–1980, 2010.

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How to design a graph with three eigenvalues

Tuesday
15h10

Gary Greaves

Nanyang Technological University, Singapore

(Joint work with Jose Yip)

Abstract

Graphs with three distinct eigenvalues are fundamental objects of study in spectral graph theory. The most well-known examples are strongly regular graphs. In 1995, Willem Haemers posed a question at the 15th British Combinatorial Conference: "Do there exist any connected graphs having three distinct eigenvalues apart from strongly regular graphs and complete bipartite graphs?"

Muzychuk and Klin initiated the study of a graph with three distinct eigenvalues via its Weisfeiler-Leman closure (also known as the coherent closure). They classified such graphs whose Weisfeiler-Leman closure has rank at most 7. In this talk, I will provide a brief overview of the history of non-regular graphs with three distinct eigenvalues, as well as present our recent results on such graphs whose Weisfeiler-Leman closure has a small rank. Our results include the discovery of a new non-regular graph with three distinct eigenvalues obtained from a quasi-symmetric design and a new conjecturally infinite family of non-regular graphs having three distinct eigenvalues obtained by switching Latin square graphs.

References

[1] Gary R.W. Greaves and Jose Yip. The coherent rank of a graph with three eigenvalues. *arXiv:2406.17395*, 36 pages, 2024.

Conditional Recurrences and 2-Quasi-Cyclic Codes

Wednesday
15h10

Emre Güday[1], Murat Şahin[2]

[1]Bilecik Seyh Edebali University - Mathematics/Faculty of Science

[2]Ankara University - Mathematics/Faculty of Science

Abstract

The construction of codes that have few weights and good parameters is an interesting problem in coding theory. For example, there is a one-to-one correspondence between 2-weight projective codes and strongly regular graphs. In addition to this, some association schemes constructed through few weight codes have a good access structure. Therefore, finding new construction method for linear codes enables to obtain new codes with few weights. In this talk focused on these problems, a new construction method for a family of linear codes, namely 2-quasi cyclic codes, is introduced. Using this method, some 1-weight AMDS codes are obtained.

This work is supported by TUBITAK (Scientific and Technological Research Council of Turkey). Project-1001, No:124F034.

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New construction of divisible design graphs using affine designs

Tuesday
15h30

Vladislav V. Kabanov

Krasovskii Institute of Mathematics and Mechanics,
Yekaterinburg, Russia

Abstract

A k-regular graph on v vertices is a *divisible design graph* if there exist integers $\lambda_1, \lambda_2, m, n$ such that the vertex set can be partitioned into m classes of size n and any two different vertices from the same class have λ_1 common neighbours, and any two vertices from different classes have λ_2 common neighbours.

Divisible design graphs were first provided by W.H. Haemers, H. Kharaghani and M. Meulenberg in [**2**]. In particular, the authors have proposed twenty constructions of divisible design graphs using various combinatorial structures. Some new combinatorial constructions of divisible design graphs were also provided in [**1,3**] and others.

In this talk, a prolific construction that produces new divisible design graphs is provided [**4**]. This construction generalizes two constructions from [**3**].

References

[1] D. Crnković, W. H. Haemers, Walk-regular divisible design graphs, *Designs, Codes and Cryptography*, **72**, 165–175, 2014.

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Bent partitions, vectorial dual-bent functions, and LP-Packings

Thursday
16h20

TEKGÜL KALAYCI

ALPEN ADRIA UNIVERSITAT KLAGENFURT-INSTITUT FÜR MATHEMATIK

(Joint work with Sezel Alkan, Nurdagül Anbar and Wilfried Meidl)

Abstract

Recently, the concept of bent partitions was introduced by Anbar and Meidl, 2022, which are partitions of elementary abelian groups having similar properties as spreads. In particular, they enable us to construct p-ary bent functions, vectorial bent functions and bent functions into finite abelian groups in general. Examples for bent partitions are generalized semifield spreads, which can be obtained from (pre)semifields with a certain additional property.

Also recently, the concept of Latin square partial difference set packings (LP-packings) of finite abelian groups was introduced by Jedwab and Li, 2022. We point out that LP-packings induce bent partitions of finite abelian groups, and generalized semifield spreads form LP-packings of elementary abelian groups. We present examples of bent partitions, which do not form LP-packings.

A vectorial bent function into an m-dimensional vector space over $\mathbb{F}_p$ is called a vectorial dual-bent function if the set of duals of its component functions forms an m-dimensional vector space of bent functions, together with the zero function. Recently, it is proven by Wang, Fu, and Wei, 2023 that vectorial dual-bent functions, under certain conditions, correspond to bent partitions with specific properties. A secondary construction of vectorial dual-bent functions, and thus of bent partitions and LP-packings, is also presented. We generalize this construction, and analyse other secondary constructions of vectorial bent functions such as the direct sum and the generalized Maiorana-McFarland construction, which also give rise to new vectorial-dual bent functions, bent partitions and LP-packings.

References

[1] S. Alkan, N. Anbar, T. Kalaycı and W. Meidl. Bent Partition, Vectorial Dual-Bent Function, and LP-Packing Constructions. *IEEE Transactions on Information Theory*, **71**: 752-767, 2025.

On higher-dimensional Hadamard matrices and designs

Monday 14h30

Vedran Krčadinac

University of Zagreb - Faculty of Science - Department of Mathematics

(Joint work with M. O. Pavčević, L. Relić and K. Tabak)

Abstract

Higher-dimensional Hadamard matrices were introduced by P. J. Shlichta in the 1970s [**4**], [**5**]. They were studied throughout the 1980s and 1990s and generalized to other types of higher-dimensional designs. The books [**1**] and [**6**] provide a fairly complete overview of the topic up to 2010. In this talk I will present some more recent results from [**2**] and [**3**]. I will also mention a forgotten success story of design theory – Room squares – and how they are generalized to higher dimensions.

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[5] PJS79 P. J. Shlichta. Higher dimensional Hadamard matrices. *IEEE Trans. Inform. Theory*, **25**:566–572, 1979.

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Exploring the presence 4^{th} MUB in the zero-entanglement subspace of C^6

Thursday
17h40

Rakesh Kumar

Indian Statistical Institute, Kolkata, India - Applied Statistics Unit

(Joint work with Saswat Sarangi and Subhamoy Maitra)

Abstract

The maximum possible number of Mutually Unbiased Bases (MUBs) in a complex Hilbert space of dimension d is $d+1$. If $d+1$ MUBs exist, then it's possible to optimally determine the state of a quantum system described by Hilbert space of dimension d [**1**]. The construction of a complete set of MUBs is well understood when the dimension d is a prime power, i.e., $d = p^k$, where p is a prime number and $k \in \mathbb{N}$ [**1**],[**2**]. However, the problem of constructing a complete set of MUBs for Hilbert spaces whose dimensions are not prime powers has been an open problem for a long time. The only known lower bound on the number of MUBs in such cases is $p^r + 1$, where p^r is the smallest prime power in the prime factorization of the dimension d. This lower bound is shown as by taking tensor products of corresponding MUBs from the prime power factors of d. In dimension 6, there are only three MUBs known till now and we have no concrete proof of presence or absence of further MUBs. In this work, we analytically prove that a fourth MUB is not possible in tensor product subspace of $d = 6$. Also, we go forward to check the presence of fourth MUB in the physically relevant, zero-entanglement subspace of $d = 6$.

References

[1] W.K. Wootters and B.D Fields. Optimal state-determination by mutually unbiased measurements. *Annals of Physics*, **191(2)**: 363-381, 1989.

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Classification of weighing matrices

Thursday
15h10

Pekka Lampio

Aalto University

(Joint work with Mikhail Ganzhinov)

Abstract

We provide a computational classification method for weighing matrices. With this method, we classify weighing matrices of orders 16, 17, 18, 19, 20 and 21 for all weights and matrices of some higher orders for some weights. The classification of weighing matrices of order 16 and weight 10 is also revised.

Boolean-Cayley-graphs: Using Sage and Python software to explore Boolean functions, their Cayley graphs and associated structures

Thursday 18h20

Paul Leopardi

The Australian National University - ACCESS-NRI

Abstract

While the author's investigations into Boolean functions, their Cayley graphs and associated structures [**5**] were inspired and encouraged by the work of Bernasconi, Codenotti [**1**], Vanderkam [**2**], Cameron [**3**], O'Cathain, Tokareva [**7**] and others, it would not have been possible without the use of Sage [**6**] and Python software. This talk describes how the author created and used the *Boolean-Cayley-graphs* software [**4**] to conduct these investigations.

References

[1] A. Bernasconi and B. Codenotti. Spectral analysis of Boolean functions as a graph eigenvalue problem. *IEEE Transactions on Computers*, 48(3):345–351, (1999).

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[7] N. Tokareva. *Bent functions: results and applications to cryptography.* Academic Press, (2015).

Partial Difference Sets: Broadening the Scope of the Denniston Family

Tuesday
17h20

Shuxing Li

University of Delaware - Department of Mathematical Sciences

(Joint work with James Davis, Sophie Huczynska, Laura Johnson, and John Polhill)

Abstract

In 1969, Denniston introduced a family of maximal arcs in Desarguesian planes of even order, a construction that gave rise to a classical family of partial difference sets with deep connections to finite geometry. These partial difference sets, later named after him, are defined within the additive group of a finite field of characteristic 2.

This naturally raises the question: Do Denniston partial difference sets exist in fields of odd characteristic? For over five decades, no progress was made on this problem—until recent breakthroughs by multiple research groups established the construction of Denniston partial difference sets in elementary abelian groups.

Building on this momentum, we extend Denniston partial difference sets to a significantly broader class of elementary abelian groups. Our construction employs character theory and relies critically on meticulous manipulation of Gauss sums over finite fields.

References

[1] S. Li, J.A. Davis, S. Huczynska, L. Johnson, and J. Polhill. Strongly regular graphs with generalized Denniston and dual generalized Denniston parameters. *Submitted*, arXiv:2501.18830v2.

On a new class of Hadamard matrices

Thursday
14h30

PETR LISONĚK

DEPARTMENT OF MATHEMATICS, SIMON FRASER UNIVERSITY,
BURNABY, BC, CANADA

(Joint work with Jasleen Phangara)

Abstract

In this talk we focus on the class of complex Hadamard matrices called S-Hadamard, which satisfy the additional condition that the elementwise product of the matrix with itself (Schur product) is also a complex Hadamard matrix. We will discuss algebraic constructions of such matrices, as well as various methods that can be employed for computational constructions. Our recently discovered parametric construction provides further insight into possible structure of these matrices. Existence results will be presented; for some matrix orders the existence question remains open. The study of these matrices is motivated by an application in quantum information theory [**1**].

References

[1] P. Lisoněk. Kochen-Specker sets and Hadamard matrices. *Theoretical Computer Science,* **800**:142–145, 2019.

New CEDFs and Related Constructions

Tuesday
17h00

Struan McCartney

University of St. Andrews - Department of Mathematics

(Joint work with Sophie Huczynska)

Abstract

External difference families were introduced in 2004 by Ogata et al. **[1]** to create secret sharing schemes with desirable properties. Circular external difference families (CEDFs) were introduced more recently in 2023 by Stinson and Veitch **[2]** as a new way to create non-malleable secret sharing schemes. I will present new constructions of infinite families and new parameters for CEDFs along with constructions for related combinatorial objects.

References

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[2] Shannon Veitch and Douglas R. Stinson. Unconditionally secure non-malleable secret sharing and circular external difference families. *Designs, Codes and Cryptography*, **92**:941-956, 2024.

Joint measurements from resolvable designs

Thursday 17h20

Daniel McNulty

University of Bari, Italy - Department of Physics

Abstract

An intrinsic feature of quantum mechanics is the impossibility to simultaneously measure non-commuting observables. However, approximations of the outcome statistics for these observables can be obtained by implementing a joint (non-projective) measurement whose marginals yield unsharp (noisy) versions of the target observables. In this talk, we explore the joint measurability properties of a physically relevant class of observables: degree–q monomials whose factors are the v (anti-commuting) generators of the Clifford algebra [**1**]. I will introduce a novel joint measurement scheme for these non-commuting observables [**2**], based on the existence of a resolvable design, i.e., a block design whose blocks form partitions of the underlying set of v elements. For quadratic monomials, we demonstrate that a resolvable design derived from an affine plane gives rise to a joint measurement that is asymptotically equivalent to the optimal joint measurement, i.e., a measurement yielding the least noisy marginals. Furthermore, we show that the existence of a $v \times v$ skew-Hadamard matrix is intimately linked to the optimality of the joint measurement. Extending these results, we show that a joint measurement from a resolvable design can simultaneously measure all degree–q monomials with sharpness asymptotically equivalent to $v^{-q/2}$, improving prior strategies by a factor proportional to q [**3**]. Finally, we apply our results to derive a necessary condition for the existence of a resolvable design.

References

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[2] D. McNulty. A graph and design theoretic approach to measurement incompatibility of binary observables. *In preparation.*

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On Alltop functions, p-ary Alltop functions and almost Hadamard matrices

Thursday
16h40

Ferruh Özbudak

Sabanci University - Faculty of Engineering and Natural Sciences

(Joint work with Fuad Hamidli and Vladimir N. Potapov)

Abstract

Alltop [**1**] introduced complex sequences utilizing cubic polynomials over $\mathbb{F}_p$ for prime $p > 3$, designed for spread spectrum radar and communication, meeting the Welch bound. Hall et al. [**2**] formally defined Alltop functions as functions with planar derivatives at every non-zero point, and discovered Alltop functions over $\mathbb{F}_{q^2}$ apart from x^3, where $q = p^n$, $p > 3$ is an odd prime. Now we classify Alltop cubic q-monomials and q-binomials over $\mathbb{F}_{q^2}$, and cubic q-monomials over $\mathbb{F}_{q^3}$ using techniques similar to [**3**]. In particular, we find all $u \in \mathbb{F}_{q^2}^*$ such that $x^3 + ux^{2q+1}$ is an Alltop function. Furthermore, we establish the non-existence of Alltop cubic q-monomials in $\mathbb{F}_{q^3}$, except x^3. We introduce the new notion of "p-ary Alltop function" $f : \mathbb{F}_{p^n} \to \mathbb{F}_p$ as a function whoose difference functions are p-ary bent functions and provide the relationship between Alltop functions and p-ary Alltop functions. We prove that $H(x, y, z) = (e^{2\pi i/p})^{f(x+y+z)-f(x+y)-f(y+z)-f(z+x)+f(x)+f(y)+f(z)}$, is a 3-dimensional almost Hadamard matrix, i. e., any 2-dimensional submatrix of H is a Hadamard matrix or all 1s matrix. We construct generalized p-ary Kerdock codes with parameters $(p^n, p^{2n+1}, (p-1)(p^{n-1} - p^{\frac{n}{2}-1})$ for even n and $(p^n, p^{2n+1}, (p-1)p^{n-1} - p^{\frac{n-1}{2}})$ for odd n based on either planar or p-ary Alltop functions. We generalize Alltop functions using higher derivatives and apply this functions to construct multidimensional almost Hadamard matrices.

References

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Additive Codes and Projective Geometries

Wednesday
15h30

Tabriz Popatia

Universidad Politécnica de Cataluña

(Joint work with Simeon Ball and Michel Lavrauw)

Abstract

In this talk, I will describe the relationship between additive codes and projective geometries, and discuss some new results we have obtained by viewing additive codes through this geometric perspective. The results of this talk can be found in [**1**] and [**2**].

Let $\mathbb{F}_q$ denote the finite field with q elements. An *additive code* of length n over $\mathbb{F}_{q^h}$ is a subset C of $(\mathbb{F}_{q^h})^n$ that is closed under addition. Such an additive code is linear over the subfield $\mathbb{F}_q$ and therefore has size q^r for some r. The notation $[n, r/h, d]_q^h$ is used to denote such an additive code with minimum distance d. We show that any $[n, r/h, d]_q^h$ additive code is equivalent to a projective h-$(n, r, d)_q$ system, which is defined as a multiset $\mathcal{S}$ of n subspaces of $PG(r-1, q)$ of dimension at most $h-1$, such that each hyperplane of $PG(r-1, q)$ contains at most $n-d$ elements of $\mathcal{S}$, and some hyperplane contains exactly $n-d$ elements.

Using this correspondence, we prove several new bounds for additive codes. In particular, we establish two analogs of the classical Griesmer bound for linear codes, extending these results to the additive case. Our Griesmer-type bound allows us to derive new restrictions on the length and dimension of additive maximum distance separable (MDS) codes. Notably, these bounds permit slightly longer codes than their linear counterparts.

Finally, we present several constructions of additive codes that leverage the relationship with projective geometries. These include families of MDS additive codes that meet our new bounds, as well as a new construction of an additive code with integral parameters that exceeds the parameters of the best-known linear codes.

References

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[2] S. Ball, and T. Popatia, Additive codes from linear codes, preprint (2025).

Close-to-perfect tensors for holographic error correction codes

Friday
12h10

Balázs Pozsgay

MTA-ELTE Momentum Integrable Quantum Dynamics Research Group
Eötvös Loránd University Budapest

(Joint work with Rafał Bistroń, Márton Mestyán and Karol Życzkowski)

Abstract

We treat special objects in design theory, which can be seen as generalizations of an Orthogonal Array (OA), or of an Absolutely Maximally Entangled state (AME). Applications in physics motivate the study of these objects, which have a weaker sense of "orthogonality" or maximal entanglement, but which also have certain symmetry properties dictated by geometry. In particular, we require orthogonality or maximal entanglement for selected subsets of columns or sites, respectively, but the resulting object should have the symmetry of a regular polygon or a regular polyherdon. We discuss selected solutions for these constraints, including new and unexpected solutions in low dimensions. We also explain the applications of these objects in lattice models of the holographic principle, where these objects are used in a tiling of the hyperbolic plane.

Based on published work **[1]** and unpublished work with the authors given above.

References

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Projection cubes of symmetric designs

Monday
14h50

Lucija Relić

University of Zagreb, Croatia

(Joint work with Vedran Krčadinac and M. O. Pavčević))

Abstract

Projection cubes of symmetric designs are n-dimensional matrices of zeros and ones such that every 2-dimensional projection is an incidence matrix of a (v, k, λ) design. We will present some basic properties and constructions of projection cubes from **[1]** and **[2]**. There is an upper bound on the dimension $n \leq v(v+1)/2$, and examples can be obtained from suitably defined higher-dimensional difference sets. A sharper upper bound $n \leq v$ holds for projection cubes constructed in this way.

References

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Weight distributions of $\mathbf{Z}_{p^s}$-linear simplex and MacDonald codes

Thursday 18h00

Sergi Sánchez-Aragón[4]

Universitat Autònoma de Barcelona - Department of Information and Communications Engineering

(Joint work with Cristina Fernández-Córdoba and Mercè Villanueva)

Abstract

The $\mathbf{Z}_{p^s}$-additive codes are subgroups of $\mathbf{Z}_{p^s}^n$ with p prime and $s \geq 1$, and can be seen as a generalization of linear codes over $\mathbf{Z}_2$, $\mathbf{Z}_4$, and $\mathbf{Z}_{2^s}$ studied extensively, such as in **[2], [3], [4]**. A $\mathbf{Z}_{p^s}$-linear code is a code over $\mathbf{Z}_p$ (not necessarily linear) which is the Gray map image of a $\mathbf{Z}_{p^s}$-additive code. We give the construction of $\mathbf{Z}_{p^s}$-linear simplex and MacDonald codes of type α and β, as a generalization of the $\mathbf{Z}_{2^s}$-linear simplex and MacDonald codes constructed and studied in **[2], [3]**. In this work, we show the fundamental parameters of these codes, as well as their complete weight distributions for the Hamming and homogeneous weights. Moreover, we show that these codes are related to the $\mathbf{Z}_{p^s}$-linear generalized Hadamard codes, studied in **[1]**.

References

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[4] This work has been partially supported by the Spanish MICINN grant PID2022-137924NB-I00, and by the Catalan AGAUR grant 2021 SGR 00643.

Legendre pairs, balanced incomplete block designs and codes

Monday
16h20

Daniel Šanko

Faculty of Mathematics, University of Rijeka, Croatia

(Joint work with Dean Crnković and Andrea Švob)

Abstract

In this work, we study connections between Legendre pairs, 2-designs and linear codes. Starting with a given Legendre pair, we construct a corresponding 2-design and span a linear code by its incidence matrix. Further, we search for 2-designs, i.e. Legendre pairs, in the constructed codes.

Complex projective 4-designs as orbits of Clifford-Weil groups

Thursday
17h00

Leonie Scheeren

RWTH Aachen University - Lehrstuhl für Algebra und Zahlentheorie

(Joint work with Hartmut Führ, Gabriele Nebe and Holger Rauhut)

Abstract

$\frac{(65-6^0)}{8} = 2 \cdot 4.$

This might have convinced you that 4 is a particularly nice number. Or you might already be convinced because you want to use complex projective t-designs in phase retrieval or for questions like distinguishing between quantum states.

For applications in low-rank matrix recovery, 4 is a particularly nice number because complex projective t-designs, with t at least 4, have been shown to be well-suited measurement schemes.[**2**] We will combine ideas from representation theory, quantum physics, invariant theory, coding theory, and numerical algebra to create a toolbox using OSCAR [**1**] that is tailored to the construction of complex projective 4-designs arising as (unions of) orbits of Clifford-Weil groups.

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On dualities of dihedral codes, generalised quaternion codes, and associated quantum codes

Friday
12h30

VÍCTOR SOTOMAYOR

UNIVERSITAT POLITÈCNICA DE VALÈNCIA - INSTITUTO UNIVERSITARIO DE MATEMÁTICA PURA Y APLICADA

(Joint work with Miguel Sales-Cabrera and Xaro Soler-Escrivà)

Abstract

A group code is a linear code that is invariant under the action of a group, so they can be viewed as (left) ideals in the corresponding group algebra. If this algebra is semisimple, then its structure is well-known due to Wedderburn's theorem. In this situation, the rich algebraic features of the group algebra help us to get a better understanding of the code.

The aim of this talk is to present recent progress concerning the algebraic structure of the Euclidean dual and the Hermitian dual of dihedral group codes and generalised quaternion group codes. As a consequence, quantum CSS codes can be constructed from these classical group codes, and in fact some optimal quantum codes can be obtained by this method.

Switching for 2-designs and applications

Andrea Švob

University of Rijeka - Faculty of Mathematics

(Joint work with Dean Crnković)

Monday
15h10

Abstract

In this talk, we introduce a switching for 2-designs. We illustrate the method by applying it to some symmetric $(64, 28, 12)$ designs. Further, we show that this type of switching can be applied to any symmetric design related to a Bush-type Hadamard matrix. By the introduced switching we construct symmetric $(36, 15, 6)$ designs leading to new Bush-type Hadamard matrices of order 36, and symmetric $(100, 45, 20)$ designs yielding Bush-type Hadamard matrices of order 100.

Circulant complex Cretan matrices

Thursday 15h30

Ondřej Turek

University of Ostrava - Department of Mathematics, Faculty of Science

(Joint work with Dardo Goyeneche, Anežka Klézlová and Daniel Uzcátegui Contreras)

Abstract

A Cretan matrix [**1**] is a square matrix S all of whose entries are real numbers of modulus not exceeding 1, at least one element in each row and column is equal to 1, and $SS^T = \omega I$ for some ω. A complex Cretan matrix is defined similarly with a natural modification: at least one element in each row and column has modulus 1, and $SS^* = \omega I$. Matrices of this type with a small number of mutually different values of entries are of particular interest; for example, real Cretan matrices whose entries take only 2 different values ("2-level Cretan matrices") generalize Hadamard matrices. In the talk, we will discuss circulant Cretan matrices. Our main focus will be Hermitian circulant Cretan matrices with complex entries whose moduli attain 2 or 3 mutually different values. We will present several constructions of those matrices, formulate conditions for their existence, and relate the new results to already known results and conjectures.

References

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Novel non-affine families of 8×8 complex Hadamard matrices

Thursday
14h50

Tuomo Valtonen

Aalto University

Abstract

Six non-affine 3-parameter families of 8×8 complex Hadamard matrices are presented. The families are mutually inequivalent as well as inequivalent to any previously known families in the literature. Each family arises from unimodular points of an algebraic variety defined by palindromic polynomials. For five of the families, the corresponding system of polynomials is solved, and bounds that guarantee unimodularity of the solutions are established.

Constructing directed strongly regular graphs using their orbit matrices and genetic algorithm

Tuesday
16h20

Tin Zrinski

Faculty of Mathematics, University of Rijeka

(Joint work with Dean Crnković and Andrea Švob)

Abstract

In this talk, we introduce orbit matrices of directed strongly regular graphs. Further, we propose a method of constructing directed strongly regular graphs with prescribed automorphism group using genetic algorithm. In the construction, we use orbit matrices, i.e. quotient matrices related to equitable partitions of adjacency matrices of putative directed strongly regular graphs induced by an action of a prescribed automorphism group. Further, we apply this method to construct some directed strongly regular graphs on 36, 52, 55 and 60 vertices.

Participants

1. **Víctor Álvarez** (Universidad de Sevilla, Spain)
2. **Andrés Armario** (Universidad de Sevilla, Spain)
3. **Santiago Barrera Acevedo** (La Trobe University, Australia)
4. **Miguel Beltrá** (Universidad de Alicante, Spain)
5. **Ingemar Bengtsson** (Stockholm University, Sweden)
6. **Anwita Bhowmik** (Hebei Normal University, Shijiazhuang, China)
7. **Patrick Browne** (University - TUS Ireland, Ireland)
8. **Wojciech Bruzda** (Center for Theoretical Physics, Polish Academy of Sciences, Poland)
9. **Oisin Campion** (University College Dublin, Ireland)
10. **Fernando Chávez** (Pontificia Universidad Católica de Chile, Chile)
11. **Robert Craigen** (University of Manitoba, Canada)
12. **Dean Crnkovic** (University of Rijeka, Croatia)
13. **Reza Dasbasteh** (University of Navarra, San Sebastian, Spain)
14. **Sara Díaz Cardell** (Unesp, Brazil)
15. **Travis Dillon** (Massachussets Institute of Technology, USA)
16. **Doris Dumičić Danilović** (University of Rijeka, Croatia)
17. **Ronan Egan** (Dublin City University, Ireland)
18. **Shalom Eliahou** (Université du Littoral Côte d'Opale, France)
19. **Raúl Falcón** (Universidad de Sevilla, Spain)

20. **Dane Flannery** (University of Galway, Ireland)
21. **María Dolores Frau** (Universidad de Sevilla, Spain)
22. **Assaf Goldberger** (Tel-Aviv University, Israel)
23. **Ana Isabel Gómez Pérez** (Universidad Rey Juan Carlos, Spain)
24. **Domingo Gómez Pérez** (Universidad de Cantabria, Spain)
25. **Manuel González Regadera** (Universidad de Sevilla, Spain)
26. **Daniel M. Gordon** (IDA Center for Communications Research - La Jolla, USA)
27. **Sergei Goryainov** (Hebei Normal University, China)
28. **Dardo Goyeneche** (Pontificia Universidad Católica de Chile, Chile)
29. **Markus Grassl** (University of Gdansk, Poland)
30. **Gary Greaves** (Nanyang Technological University, Singapore)
31. **Emre Güday** (Bilecik Seyh Edebali University, Turkiye)
32. **Félix Gudiel** (Universidad de Sevilla, Spain)
33. **María Belén Güemes** (Universidad de Sevilla, Spain)
34. **Jonathan Jedwab** (Simon Fraser University, Canada)
35. **Vladislav Kabanov** (N.N. Krasovskii Institute, Russia)
36. **Tekgül Kalaycı** (Alpen Adria Universitat Klagenfurt, Germany)
37. **Hadi Kharaghani** (University Of Lethbridge, Canada)
38. **Anezka Klezlova** (University of Ostrava, Czech Republic)
39. **Ilias Kotsireas** (Wilfrid Laurier University, Canada)
40. **Vedran Krcadinac** (University of Zagreb, Croatia)
41. **Rakesh Kumar** (Indian Statistical Institute, Kolkata, India)
42. **Pekka Lampio** (Aalto University, Finland)
43. **Paul Leopardi** (The Australian National University, Australia)
44. **Shuxing Li** (University of Delaware, USA)

45. **Petr Lisonek** (Simon Fraser University, Canada)
46. **Gary MacConnell** (Imperial College, UK)
47. **Máte Matolcsi** (Renyi Institute of Mathematics, Hungary)
48. **Struan McCartney** (University of St. Andrews, Scotland)
49. **Daniel McNulty** (Universitá di Bari, Italy)
50. **Koji Momihara** (Kumamoto University, Japan)
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55. **Balázs Pozsgay** (Eötvös Loránd University Budapest, Hungary)
56. **Lucija Relic** (University of Zagreb, Croatia)
57. **Verónica Requena** (Universidad de Alicante, Spain)
58. **Murat Sahin** (Ankara University, Turkiye)
59. **Sergi Sánchez** (Universidad Autónoma de Barcelona, Spain)
60. **Daniel Sanko** (University of Rijeka, Croatia)
61. **Leonie Scheeren** (RWTH Aachen University, Germany)
62. **Jennifer Seberry** (University of Wollongong, Australia)
63. **Víctor Sotomayor** (Universidad Politécnica de Valencia, Spain)
64. **Sho Suda** (National Defense Academy of Japan, Japan)
65. **Andrea Svob** (University of Rijeka, Croatia)
66. **Ferenc Szollosi** (Shimane University, Japan)
67. **Stefan Trandafir** (Universidad de Sevilla, Spain)
68. **Ondrej Turek** (University of Ostrava, Czech Republic)
69. **Tuomo Valtonen** (Aalto university, Finland)

70. **Mercé Villanueva** (Universidad Autónoma de Barcelona, Spain)
71. **Stefan Weigert** (University of York, UK)
72. **Mihaly Weiner** (Institute of Mathematics, BME, Hungary)
73. **Qing Xian** (Southern University of Science and Technology, China)
74. **Tin Zrinski** (University of Rijeka, Croatia)
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