

Cyclic Relative Difference Sets

DANIEL M. GORDON

IDA CENTER FOR COMMUNICATIONS RESEARCH - LA JOLLA

Abstract

Let G be a group of order mn with a normal subgroup N of order n . An (m, n, k, λ) -relative difference set (RDS) of G relative to N is a set R of k elements of G such that every element of $G \setminus N$ occurs exactly λ times as a difference of distinct elements of R , i.e. in the group ring $\mathbb{Z}[G]$, R satisfies:

$$RR^{-1} = k + \lambda(G - N).$$

Any relative difference set is a lifting of an $(m, k, n\lambda)$ -difference set in G/N . All known nontrivial difference sets with liftings to a cyclic RDS have parameters of complements of classical Singer difference sets:

$$\left(\frac{q^{d+1} - 1}{q - 1}, q^d, q^{d-1}(q - 1) \right).$$

In this talk we give new evidence supporting the conjecture that these are the only ones, and discuss connections to circulant weighing matrices.