The maximum number of Desarguesian mutually orthogoval affine planes of order 2^n

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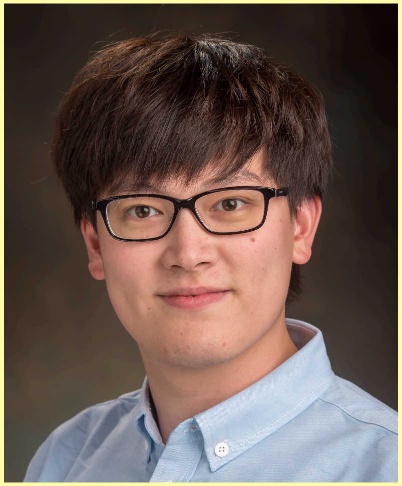

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27 May 2025

Outline

- Desarguesian affine plane
- Orthogoval
- Central question
- Randomised search
- Explicit construction
- Deterministic search
- Feasible subspaces
- Future research

Desarguesian affine plane

- An affine plane of prime power order q is a 2-$(q^2, q, 1)$ design

- Desarguesian affine plane AG$(2, q)$ of order q is unique up to isomorphism

Girard Desargues 1591–1661

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Affine plane $\mathrm{AG}(2, 8)$

- Construct $\mathbb{F}_8$ from primitive polynomial $f(x) = x^3 + x + 1$ with root α

- The nine 1-dim subspaces of 2-dim vector space $\mathbb{F}_8 \times \mathbb{F}_8$ are

  $\langle(0,1)\rangle$ $\langle(1,0)\rangle$ $\langle(1,1)\rangle$ $\langle(1,\alpha)\rangle$ $\langle(1,\alpha^2)\rangle$

  $\langle(1,\alpha^3)\rangle$ $\langle(1,\alpha^4)\rangle$ $\langle(1,\alpha^5)\rangle$ $\langle(1,\alpha^6)\rangle$

- Map each 1-dim subspace of $\mathbb{F}_8 \times \mathbb{F}_8$ to 3-dim subspace of $\mathbb{F}_2^6$

Affine plane AG(2, 8)

- Map each 1-dim subspace of $\mathbb{F}_8 \times \mathbb{F}_8$ to 3-dim subspace of $\mathbb{F}_2^6$
  - ⋆ elements of 1-dim subspace $\langle(1,\alpha^2)\rangle$ are

$$
\begin{array}{lll}
(0,0) & \mapsto & \boxed{(000\ 000)} \\
(1,\alpha^2) & \mapsto & (001\ 100) \\
(\alpha,\alpha^3) & \mapsto & (010\ 011) \\
(\alpha^2,\alpha^4) & \mapsto & (100\ 110)
\end{array}
\qquad
\begin{array}{lll}
(\alpha^3,\alpha^5) & \mapsto & (011\ 111) \\
(\alpha^4,\alpha^6) & \mapsto & (110\ 101) \\
(\alpha^5,1) & \mapsto & (111\ 001) \\
(\alpha^6,\alpha) & \mapsto & (101\ 010)
\end{array}
$$

$$f(x) = x^3 + x + 1$$

  - ⋆ decimal representation of 3-dim subspace is

0	12	19	38	31	53	57	42

Affine plane $\mathrm{AG}(2,8)$

- Map each 1-dim subspace of $\mathbb{F}_8 \times \mathbb{F}_8$ to 3-dim subspace of $\mathbb{F}_2^6$

$\langle(0,1)\rangle \mapsto$	0	1	2	4	3	6	7	5
$\langle(1,0)\rangle \mapsto$	0	8	16	32	64	48	56	40
$\langle(1,1)\rangle \mapsto$	0	9	18	36	27	54	63	45
$\langle(1,\alpha)\rangle \mapsto$	0	10	20	35	30	55	61	41
$\langle(1,\alpha^2)\rangle \mapsto$	0	12	19	38	31	53	57	42
$\langle(1,\alpha^3)\rangle \mapsto$	0	11	22	39	29	49	58	44
$\langle(1,\alpha^4)\rangle \mapsto$	0	14	23	37	25	50	60	43
$\langle(1,\alpha^5)\rangle \mapsto$	0	15	21	33	26	52	59	46
$\langle(1,\alpha^6)\rangle \mapsto$	0	13	17	34	28	51	62	47

$$\text{spread in } \mathbb{F}_2^6 \xrightarrow{\text{additive cosets}} \mathrm{AG}(2,8)$$

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Affine plane $\mathrm{AG}(2, 8)$

- Map each 1-dim subspace of $\mathbb{F}_8 \times \mathbb{F}_8$ to 3-dim subspace of $\mathbb{F}_2^6$
- The nine 3-dim subspaces of $\mathbb{F}_2^6$ form a spread in $\mathbb{F}_2^6$
  - ⋆ each pair of subspaces intersects only in 0
  - ⋆ the nonzero elements of the subspaces partition the nonzero elements of $\mathbb{F}_2^6$
- Affine plane $\mathrm{AG}(2, 8)$ comprises all 8 additive cosets in $\mathbb{F}_2^6$ of each subspace of the spread
  - ⋆ 72 subspaces (lines) each containing 8 points of $\mathbb{F}_2^6$
  - ⋆ every pair of points occurs in exactly 1 line

Orthogoval sets of lines

0	1	2	3	4	5	6	7
0	8	16	24	32	40	48	56
0	9	18	27	36	45	54	63
0	10	20	30	35	41	55	61
0	11	22	29	39	44	49	58
0	12	19	31	38	42	53	57
0	13	17	28	34	47	51	62
0	14	23	25	37	43	50	60
0	15	21	26	33	46	52	59

spread C in $\mathbb{F}_2^6$

$\downarrow$

$AG(2, 8)$

$\longleftrightarrow$

orthogoval affine planes

0	1	8	9	20	21	28	29
0	2	12	14	32	34	44	46
0	3	24	27	37	38	61	62
0	4	19	23	35	39	48	52
0	5	26	31	40	45	50	55
0	6	16	22	41	47	57	63
0	7	10	13	49	54	59	60
0	11	18	25	33	42	51	56
0	15	17	30	36	43	53	58

spread S in $\mathbb{F}_2^6$

$\downarrow$

$AG(2, 8)$

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Orthogoval sets of lines

- Orthogoval sets of lines: each line of one set intersects each line of other set in at most 2 points

- Orthogoval spreads extend to orthogoval Desarguesian affine planes when take additive cosets
  - ⋆ simple consequence of subspace property
  - ⋆ use to construct perfect hash families, covering arrays

- Orthogoval affine planes: each line of one plane is an oval in the other

Orthogoval sets of lines

1	2	3	4	5	6	7
8	16	24	32	40	48	56
9	18	27	36	45	54	63
10	20	30	35	41	55	61
11	22	29	39	44	49	58
12	19	31	38	42	53	57
13	17	28	34	47	51	62
14	23	25	37	43	50	60
15	21	26	33	46	52	59

spread C in $\mathbb{F}_2^6$

orthogoval spreads

1	8	9	20	21	28	29
2	12	14	32	34	44	46
3	24	27	37	38	61	62
4	19	23	35	39	48	52
5	26	31	40	45	50	55
6	16	22	41	47	57	63
7	10	13	49	54	59	60
11	18	25	33	42	51	56
15	17	30	36	43	53	58

spread S in $\mathbb{F}_2^6$

- Orthogoval spreads C and S : each line of one spread intersects each line of other spread in at most 2 points
  - ⋆ ignore zero element in each line: then intersection is in at most 1 point

Mutually orthogoval

1	2	3	4	5	6	7
8	16	24	32	40	48	56
9	18	27	36	45	54	63
10	20	30	35	41	55	61
11	22	29	39	44	49	58
12	19	31	38	42	53	57
13	17	28	34	47	51	62
14	23	25	37	43	50	60
15	21	26	33	46	52	59

1	8	9	20	21	28	29
2	12	14	32	34	44	46
3	24	27	37	38	61	62
4	19	23	35	39	48	52
5	26	31	40	45	50	55
6	16	22	41	47	57	63
7	10	13	49	54	59	60
11	18	25	33	42	51	56
15	17	30	36	43	53	58

1	12	13	22	23	26	27
2	28	30	40	42	52	54
3	17	18	32	35	49	50
4	11	15	34	38	41	45
5	24	29	33	36	57	60
6	8	14	51	53	59	61
7	19	20	43	44	56	63
9	16	25	39	46	55	62
10	21	31	37	47	48	58

spread in $\mathbb{F}_2^6$

$\downarrow$

$AG(2, 8)$

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Central Question

- Colbourn, Ingalls, Jedwab, Saaltink, Smith, Stevens 2024 asked:

What is the largest number of mutually orthogoval Desarguesian affine planes of order $q = 2^n$?

- Equivalently:

What is the largest number of mutually orthogoval spreads in $\mathbb{F}_2^{2n}$?

Finding orthogoval spreads in $\mathbb{F}_2^6$

1	2	3	4	5	6	7
8	16	24	32	40	48	56
9	18	27	36	45	54	63
10	20	30	35	41	55	61
11	22	29	39	44	49	58
12	19	31	38	42	53	57
13	17	28	34	47	51	62
14	23	25	37	43	50	60
15	21	26	33	46	52	59

spread C in $\mathbb{F}_2^6$

$$\underbrace{\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}_{1} = \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}}_{61}$$

3	24	27	37	38	61	62
7	19	20	43	44	56	63
6	9	15	16	22	25	31
1	28	29	40	41	52	53
4	17	21	35	39	50	54
13	23	26	32	45	55	58
12	18	30	34	46	48	60
5	11	14	33	36	42	47
2	8	10	49	51	57	59

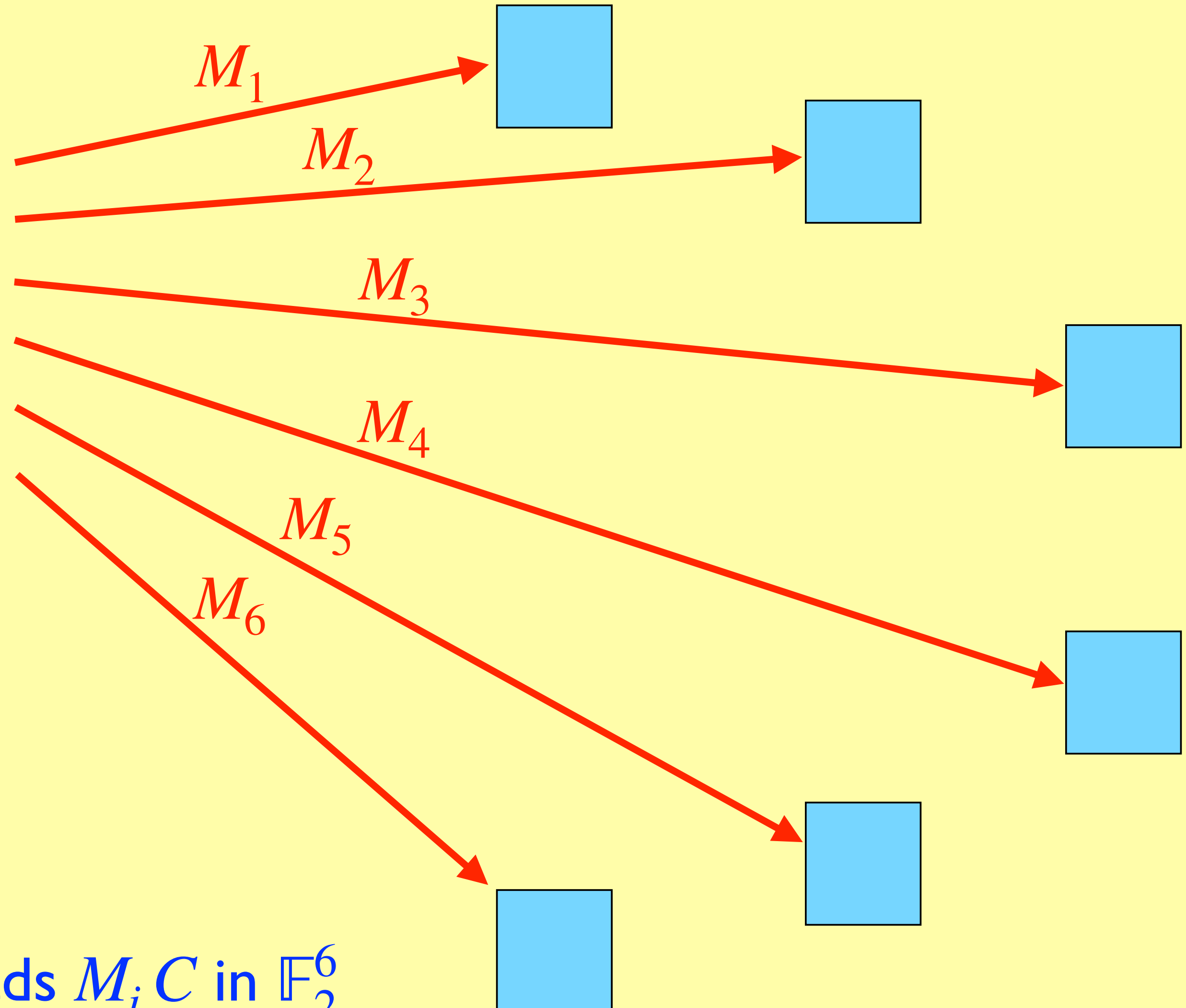
spread MC in $\mathbb{F}_2^6$

- (Colbourn et al 2024). Choose many $M \in \mathrm{GL}(6, \mathbb{F}_2)$ randomly, retain those giving spread MC orthogoval to C

Finding orthogoval spreads in $\mathbb{F}_2^6$

1	2	3	4	5	6	7
8	16	24	32	40	48	56
9	18	27	36	45	54	63
10	20	30	35	41	55	61
11	22	29	39	44	49	58
12	19	31	38	42	53	57
13	17	28	34	47	51	62
14	23	25	37	43	50	60
15	21	26	33	46	52	59

spread C in $\mathbb{F}_2^6$



spreads $M_i C$ in $\mathbb{F}_2^6$

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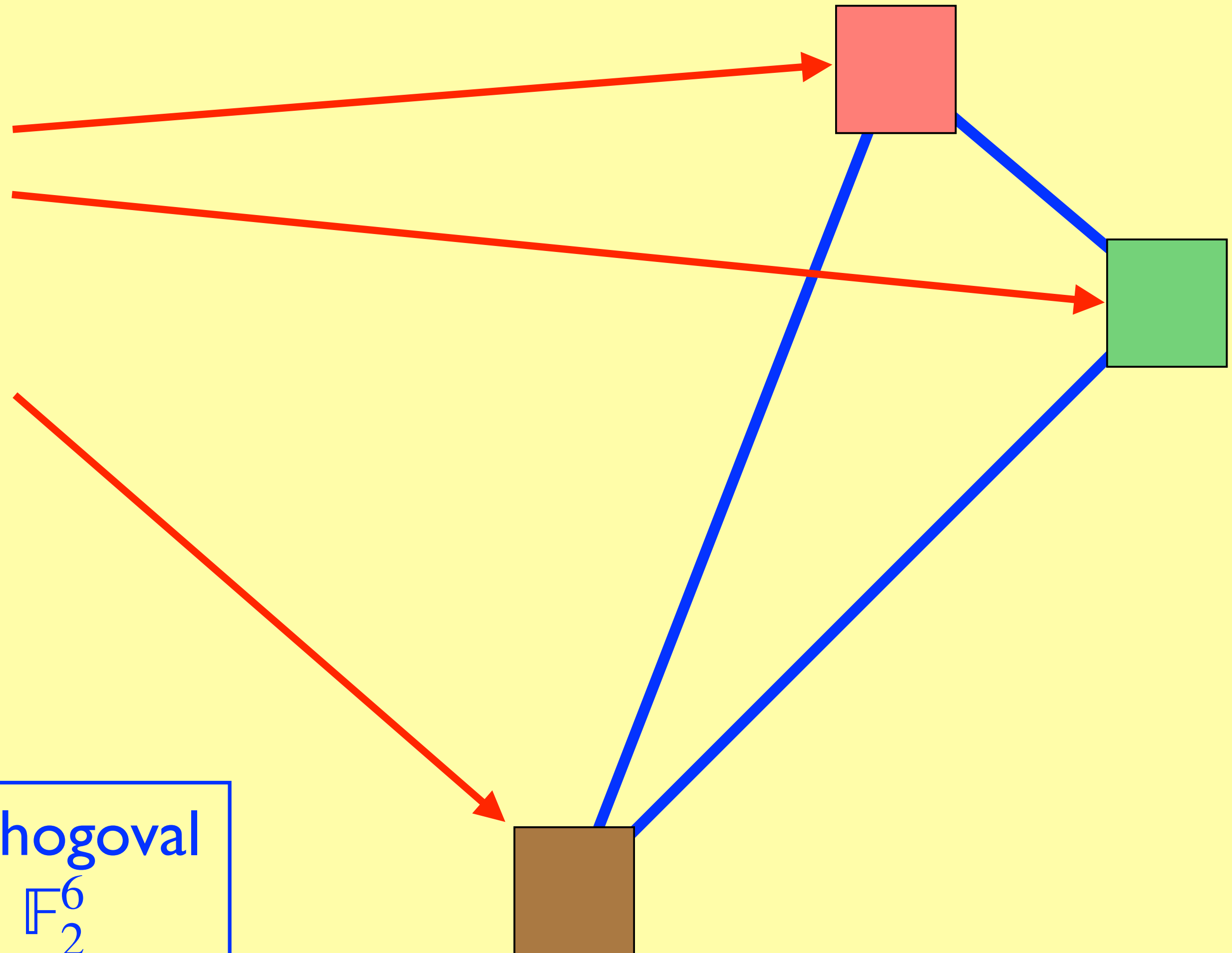
Finding orthogoval spreads in $\mathbb{F}_2^6$

1	2	3	4	5	6	7
8	16	24	32	40	48	56
9	18	27	36	45	54	63
10	20	30	35	41	55	61
11	22	29	39	44	49	58
12	19	31	38	42	53	57
13	17	28	34	47	51	62
14	23	25	37	43	50	60
15	21	26	33	46	52	59

spread C in $\mathbb{F}_2^6$



4 mutually orthogoval spreads in $\mathbb{F}_2^6$

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Finding orthogoval spreads in $\mathbb{F}_2^6$

1	2	3	4	5	6	7
8	16	24	32	40	48	56
9	18	27	36	45	54	63
10	20	30	35	41	55	61
11	22	29	39	44	49	58
12	19	31	38	42	53	57
13	17	28	34	47	51	62
14	23	25	37	43	50	60
15	21	26	33	46	52	59

spread C in $\mathbb{F}_2^6$

$$\underbrace{\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}_{1} = \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}}_{61}$$

3	24	27	37	38	61	62
7	19	20	43	44	56	63
6	9	15	16	22	25	31
1	28	29	40	41	52	53
4	17	21	35	39	50	54
13	23	26	32	45	55	58
12	18	30	34	46	48	60
5	11	14	33	36	42	47
2	8	10	49	51	57	59

spread MC in $\mathbb{F}_2^6$

- (Colbourn et al 2024). Choose many $M \in \mathrm{GL}(6, \mathbb{F}_2)$ randomly, retain those giving spread MC orthogoval to C
  - ⋆ find a largest clique in associated graph

mutually orthogoval spreads in $\mathbb{F}_2^{2n}$

n	order q of affine plane	known # mutually orthogoval spreads	method	maximum?
2	4	7	randomised search	Y
3	8	7	randomised search	?
4	16			
5	32			
6	64			
n	2^n			
n (coprime to 6)	2^n			

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Known constructions

- Theorem (Colbourn et al 2024). There are
  - ⋆ 2 (mutually) orthogoval spreads in $\mathbb{F}_2^{2n}$ for all n
  - ⋆ 3 mutually orthogoval spreads in $\mathbb{F}_2^{2n}$ for n coprime to 6
  - ⋆ by explicit construction, using finite geometry and analysis of the roots of specific polynomials

mutually orthogoval spreads in $\mathbb{F}_2^{2n}$

n	order q of affine plane	known # mutually orthogoval spreads	method	maximum?
2	4	7	randomised search	Y
3	8	7	randomised search	?
4	16	2	construction	?
5	32	3	construction	?
6	64	2	construction	?
n	2^n	2	construction	?
n (coprime to 6)	2^n	3	construction	?

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Known constructions

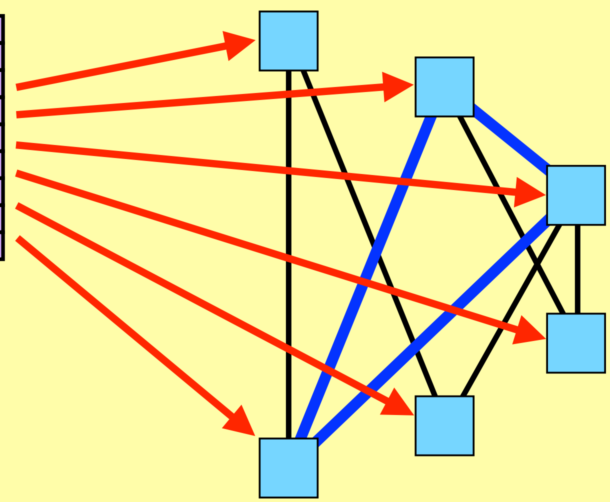
- Theorem (Colbourn et al 2024). There are
  - ⋆ 2 (mutually) orthogoval spreads in $\mathbb{F}_2^{2n}$ for all n
  - ⋆ 3 mutually orthogoval spreads in $\mathbb{F}_2^{2n}$ for n coprime to 6
  - ⋆ by explicit construction, using finite geometry and analysis of the roots of specific polynomials

Can these results be improved ?

Deterministic search

- Start with the canonical spread C in $\mathbb{F}_2^6$ (order $q = 8$)
- Construct explicitly all other 1904639 distinct spreads in $\mathbb{F}_2^6$
- Of these, only 3864 spreads are orthogoval to C
- A largest clique in the graph on 3864 vertices has size 6
- So largest number of mutually orthogoval spreads in $\mathbb{F}_2^6$ is 7

1	2	3	4	5	6	7
8	16	24	32	40	48	56
9	18	27	36	45	54	63
10	20	30	35	41	55	61
11	22	29	39	44	49	58
12	19	31	38	42	53	57
13	17	28	34	47	51	62
14	23	25	37	43	50	60
15	21	26	33	46	52	59



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mutually orthogoval spreads in $\mathbb{F}_2^{2n}$

n	order q of affine plane	known # mutually orthogoval spreads	method	maximum?
2	4	7	randomised search	Y
3	8	7	randomised search	$\boxed{\text{Y}}$
4	16	2	construction	?
5	32	3	construction	?
6	64	2	construction	?
n	2^n	2	construction	?
n (coprime to 6)	2^n	3	construction	?

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Deterministic search

- Start with the canonical spread C in $\mathbb{F}_2^6$ (order $q = 8$)
- Construct explicitly all other 1904639 distinct spreads in $\mathbb{F}_2^6$
- Of these, only 3864 spreads are orthogoval to C
- A largest clique in the graph on 3864 vertices has size 6
- So largest number of mutually orthogoval spreads in $\mathbb{F}_2^6$ is 7
- This method solves order $q = 8$, but for order $q = 16$ there are $21\,846\,665\,723\,904$ distinct spreads in $\mathbb{F}_2^8$ 😱

Feasible subspaces

1	2	3	4	5	6	7
8	16	24	32	40	48	56
9	18	27	36	45	54	63
10	20	30	35	41	55	61
11	22	29	39	44	49	58
12	19	31	38	42	53	57
13	17	28	34	47	51	62
14	23	25	37	43	50	60
15	21	26	33	46	52	59

spread C in $\mathbb{F}_2^6$

orthogoval spreads

1	8	9	20	21	28	29
2	12	14	32	34	44	46
3	24	27	37	38	61	62
4	19	23	35	39	48	52
5	26	31	40	45	50	55
6	16	22	41	47	57	63
7	10	13	49	54	59	60
11	18	25	33	42	51	56
15	17	30	36	43	53	58

spread S in $\mathbb{F}_2^6$

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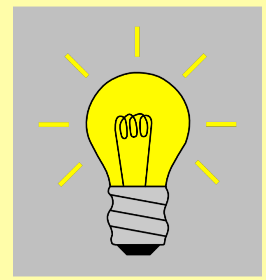
Feasible subspaces

1	2	3	4	5	6	7
8	16	24	32	40	48	56
9	18	27	36	45	54	63
10	20	30	35	41	55	61
11	22	29	39	44	49	58
12	19	31	38	42	53	57
13	17	28	34	47	51	62
14	23	25	37	43	50	60
15	21	26	33	46	52	59

spread C in $\mathbb{F}_2^6$

1	8	9	20	21	28	29

feasible subspace in $\mathbb{F}_2^6$



Find all feasible subspaces individually, then combine into orthogoval spreads!

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Feasible subspaces

- Feasible subspace intersects in at most one (nonzero) element with each subspace of the canonical spread

- How to find all feasible n-dimensional subspaces ?
  - ⋆ silly: filter all n-dimensional subspaces of $\mathbb{F}_2^{2n}$ directly
  - ⋆ smart: construct the feasible n-dimensional subspaces recursively from feasible lower-dimensional subspaces

Feasible subspaces

1	2	3	4	5	6	7
$\boxed{8}$	$\boxed{16}$	$\boxed{24}$	32	40	48	56
9	18	27	36	45	54	63
10	20	30	35	41	55	61
11	22	29	39	44	49	58
12	19	31	38	42	53	57
13	17	28	34	47	51	62
14	23	25	37	43	50	60
15	21	26	33	46	52	59

1	8	9	20	21	28	29

$$\begin{array}{ccc} 1 & \boxed{8} & 9 \end{array}$$

$$\oplus \quad \boxed{16} \quad = \quad 17 \quad \boxed{24} \quad 25$$

~~10~~	~~11~~	~~12~~	~~13~~	~~14~~	~~15~~	~~16~~	~~17~~	18	19	20	21	22	23
~~24~~	~~25~~	26	27	28	29	30	31	32	33	34	35	36	37
38	39	40	41	42	43	44	45	46	47	48	49	50	51
52	53	54	55	56	57	58	59	60	61	62	63		

- Extend each feasible 2-dimensional subspace T to all feasible 3-dimensional subspaces whose smallest elements are T

Feasible subspaces

1	2	3	4	5	6	7
8	16	24	32	40	48	56
9	18	27	36	45	54	63
10	20	30	35	41	55	61
11	22	29	39	44	49	58
12	19	31	38	42	53	57
13	17	28	34	47	51	62
14	23	25	37	43	50	60
15	21	26	33	46	52	59

1	8	9	20	21	28	29

$$1 \quad 8 \quad \boxed{9}$$

$$\oplus \quad \boxed{18} \quad = \quad 19 \quad 26 \quad \boxed{27}$$

~~10~~	~~11~~	~~12~~	~~13~~	~~14~~	~~15~~	~~16~~	~~17~~	~~18~~	~~19~~	20	21	22	23
~~24~~	~~25~~	~~26~~	~~27~~	28	29	30	31	32	33	34	35	36	37
38	39	40	41	42	43	44	45	46	47	48	49	50	51
52	53	54	55	56	57	58	59	60	61	62	63		

- Extend each feasible 2-dimensional subspace T to all feasible 3-dimensional subspaces whose smallest elements are T

Feasible subspaces

1	2	3	4	5	6	7
8	16	24	32	40	48	56
9	18	27	36	45	54	63
10	20	30	35	41	55	61
11	22	29	39	44	49	58
12	19	31	38	42	53	57
13	17	28	34	47	51	62
14	23	25	37	43	50	60
15	21	26	33	46	52	59

1	8	9	20	21	28	29

1 8 9

$\oplus$ 20 = 21 28 29

1	8	9	20	21	28	29

~~10~~ ~~11~~ ~~12~~ ~~13~~ ~~14~~ ~~15~~ ~~16~~ ~~17~~ ~~18~~ ~~19~~ ~~20~~ ~~21~~ 22 23
~~24~~ ~~25~~ ~~26~~ ~~27~~ ~~28~~ ~~29~~ 30 31 32 33 34 35 36 37
38 39 40 41 42 43 44 45 46 47 48 49 50 51
52 53 54 55 56 57 58 59 60 61 62 63

- Extend each feasible 2-dimensional subspace T to all feasible 3-dimensional subspaces whose smallest elements are T

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Feasible subspaces

1	2	3	4	5	6	7
8	16	24	32	40	48	56
9	18	27	36	45	54	63
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11	22	29	39	44	49	58
12	19	31	38	42	53	57
13	17	28	34	47	51	62
14	23	25	37	43	50	60
15	21	26	33	46	52	59

1 8 9 20 21 28 29

1 8 9

$\oplus$ 22 = 23 30 31

1 8 9 22 23 30 31

~~10~~ ~~11~~ ~~12~~ ~~13~~ ~~14~~ ~~15~~ ~~16~~ ~~17~~ ~~18~~ ~~19~~ ~~20~~ ~~21~~ ~~22~~ ~~23~~
~~24~~ ~~25~~ ~~26~~ ~~27~~ ~~28~~ ~~29~~ ~~30~~ ~~31~~ 32 33 34 35 36 37
38 39 40 41 42 43 44 45 46 47 48 49 50 51
52 53 54 55 56 57 58 59 60 61 62 63

- Extend each feasible 2-dimensional subspace *T* to all feasible 3-dimensional subspaces whose smallest elements are *T*

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Feasible subspaces

1	2	3	4	5	6	7
8	16	24	32	40	48	56
9	18	27	36	45	54	63
10	20	30	35	41	55	61
11	22	29	39	44	49	58
12	19	31	38	42	53	57
13	17	28	34	47	51	62
14	23	25	37	43	50	60
15	21	26	33	46	52	59

spread C

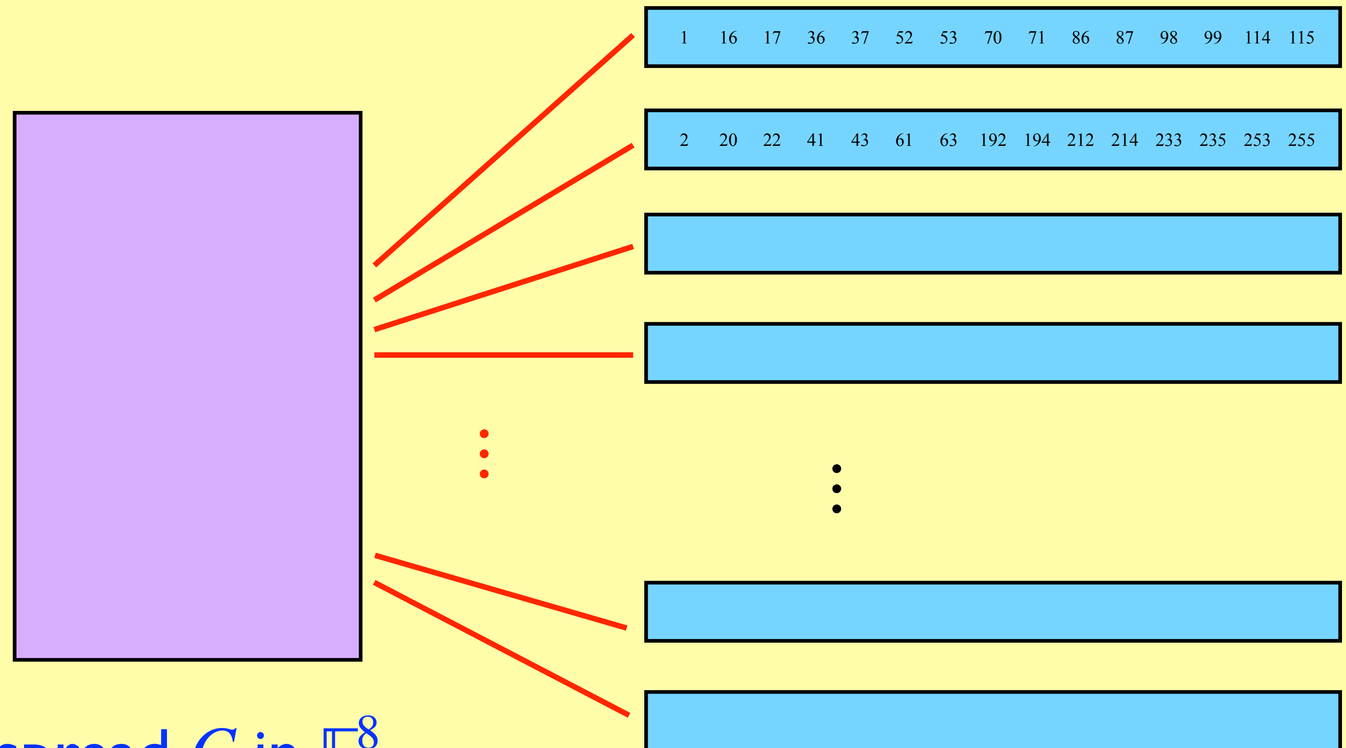
1	8	9	20	21	28	29
1	8	9	22	23	30	31
1	8	9	34	35	42	43
1	8	9	38	39	46	47
1	8	9	50	51	58	59
1	8	9	52	53	60	61

- There are six 3-dimensional subspaces that are feasible wrt spread C and in which $1 \quad 8 \quad 9$ are the smallest elements

Feasible subspaces in $\mathbb{F}_2^8$

- Start with the canonical spread C in $\mathbb{F}_2^8$ (order $q = 16$)
- There are $21\,846\,665\,723\,904$ distinct spreads in $\mathbb{F}_2^8$
- But there are only $200\,787$ 4-dimensional subspaces of $\mathbb{F}_2^8$
- Of these, only 4080 are feasible wrt to C

Finding orthogoval spreads in $\mathbb{F}_2^8$



spread C in $\mathbb{F}_2^8$

4080 feasible subspaces in $\mathbb{F}_2^8$

Find all 17-subsets of the feasible subspaces containing each element of $\{1,2,\ldots,255\}$ exactly once

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Feasible subspaces in $\mathbb{F}_2^8$

- Start with the canonical spread C in $\mathbb{F}_2^8$ (order $q = 16$)
- There are $21\,846\,665\,723\,904$ distinct spreads in $\mathbb{F}_2^8$
- But there are only $200\,787$ 4-dimensional subspaces of $\mathbb{F}_2^8$
- Of these, only 4080 are feasible wrt to C
- Exact cover search finds all 17-subsets of the 4080 feasible subspaces of $\mathbb{F}_2^8$ that give an orthogoval spread
- Gives that there are $40\,800$ spreads orthogoval to C
- No two of the $40\,800$ spreads are orthogoval to each other
- So largest # mutually orthogoval spreads in $\mathbb{F}_2^8$ is 2

mutually orthogoval spreads in $\mathbb{F}_2^{2n}$

n	order q of affine plane	known # mutually orthogoval spreads	method	maximum?
2	4	7	randomised search	Y
3	8	7	randomised search	Y
4	16	2	construction	$\boxed{\textbf{Y}}$
5	32	3	construction	?
6	64	2	construction	?
n	2^n	2	construction	?
n (coprime to 6)	2^n	3	construction	?

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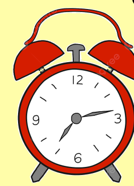
Feasible subspaces in $\mathbb{F}_2^{10}$

- What about order $q = 32$?
- There are $109\,221\,651$ 5-dimensional subspaces of $\mathbb{F}_2^{10}$
- Of these, only $65\,472$ are feasible wrt to C (4 seconds)
- Exact cover search to find all 33-subsets of the $65\,472$ feasible subspaces of $\mathbb{F}_2^{10}$ that give orthogoval spread
  - ⋆ estimated run time 3 months 
- Partial results suggest no improvement to known construction for $q = 32$
  - ⋆ nor to known construction for $q = 64$

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Future research

- Use structure of solutions for orders $4, 8, 16$ to reduce search time for order 32 and bring order 64 within reach

- Assuming the known constructions are optimal, what might a proof depend on ?

- Extend to Desarguesian affine planes in odd characteristic