Exact SIC-POVMs from Permutation Symmetries

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Abstract

A SIC-POVM corresponds to a set of d^2 complex equiangular lines in dimension d. More than 25 years ago, Gerhard Zauner conjectured that SIC-POVMs exist in all dimensions. Despite significant progress, a proof of their existence for infinitely many dimensions has yet to be found.

Initially, solutions were obtained using numerical optimization or rather complex Gröbner basis calculations [4]. There is a general approach that, based on some conjectures, allows to convert numerical solutions into exact ones [1]. That approach is based on the corresponding rank-one projection operator with $O(d^2)$ parameters. We show that in certain dimensions, permutation symmetries allow to directly deduce an exact so-called fiducial vector with $O(d)$ parameters. This technique has enabled us to convert numerical solutions from Stark units [2] as well as from the approach in [3] into exact ones.

References

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