Intrinsic volumes of the quantum state space and mutually unbiased bases

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(Joint work with Zs. Szilágyi)

Abstract

Previous studies about the geometrical properties of the state space of a d-level quantum system have determined its volume and surface area; see [1]. Building on this foundation, we derive explicit formulas [2] for two additional intrinsic volume quantities.

The question of whether a complete set of mutually unbiased bases exists in dimension d can be equivalently framed as whether a specific convex polytope can be inscribed within the state space of a d-level quantum system [3]. One motivation for our work was the hypothesis that a smaller intrinsic volume of the state space compared to the corresponding intrinsic volume of the mentioned polytope could rule out such an inscription.

While our computations of these two intrinsic volumes do not lead to this conclusion, they nonetheless provide fundamental insights into the geometric structure of quantum state spaces. In particular, we show that these quantities can be used to rule out the existence of some unit-vector "configuration" (though not the one formed by the bases vectors of a complete set of mutually unbiased bases).

References

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