Close-to-perfect tensors for holographic error correction codes

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Eötvös Loránd University, Budapest

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Talk based on

- M. Mestyán, BP, I. Wanless, *SciPost Phys. 16, 010 (2024)*
- R. Bistron, M. Mestyán, BP, K. Życzkowski *(in preparation)*

Perfect tensors

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- AME(N,D) element of $\bigotimes_{j=1}^{N} \mathbb{C}^D$ that has maximal bi-partite entanglement for all bi-partitions

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Imperfect tensors? Close-to-perfect tensors?

- Large, but not maximal entanglement for all bi-partitions
- Maximal entanglement only for selected bi-partitions

Why?

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Applications: Tensor networks

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- Dual unitary operators (for 1D quantum cellular automata)
- Holographic error correcting codes

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Close-to-perfect tensors

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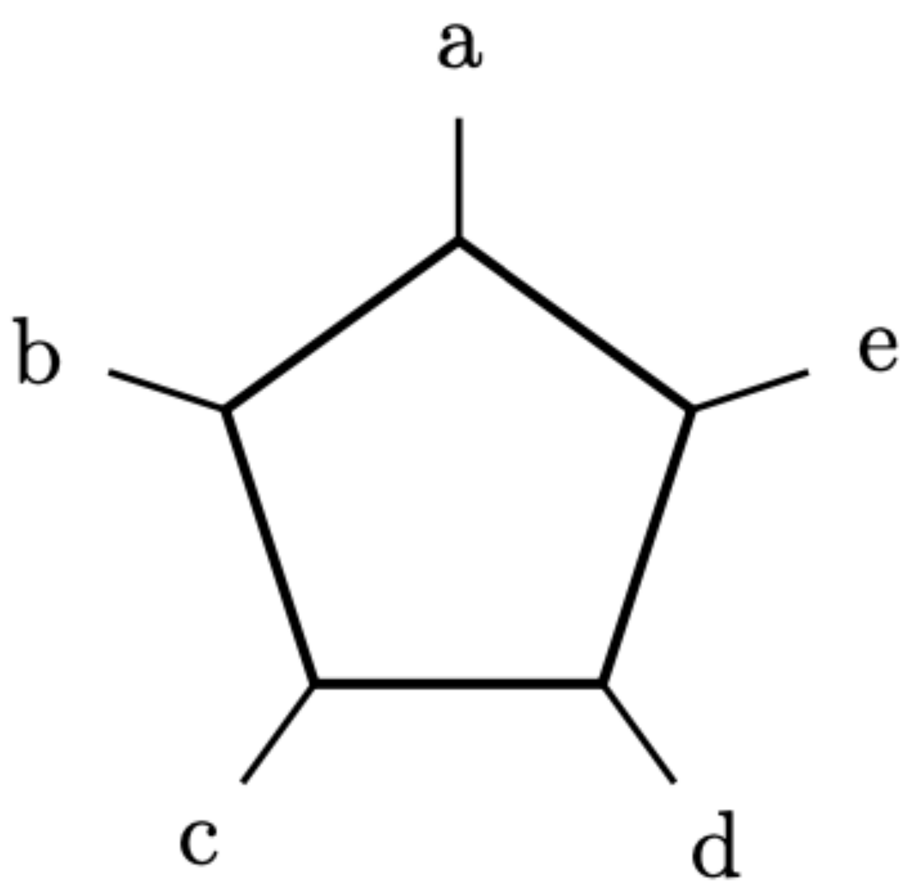
General framework: Using graphs to encode the bi-partitions

Planar maximally entangled states of 5 qubits



- $\left|\Psi\right\rangle$ is an element of $\otimes_{j=1}^{5} \mathbb{C}^2$
- Components $\Psi_{a,b,c,d,e}$

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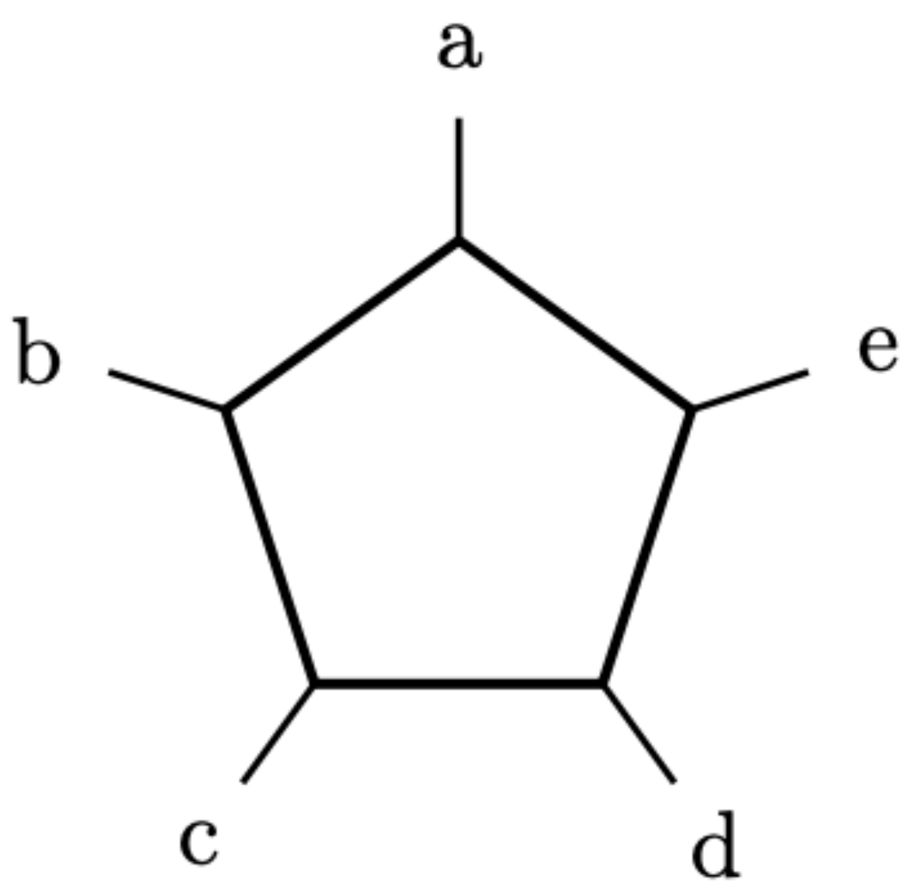

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Requirements:

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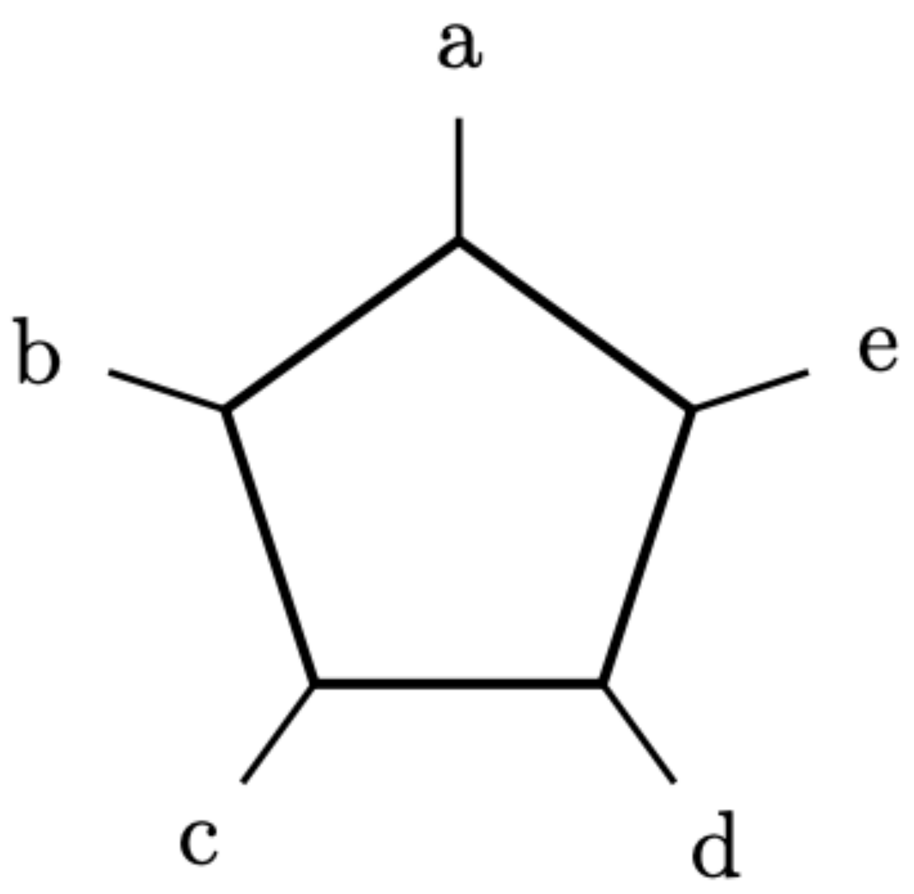

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- Pentagonal symmetry
- Maximal entanglement for neighbouring pairs

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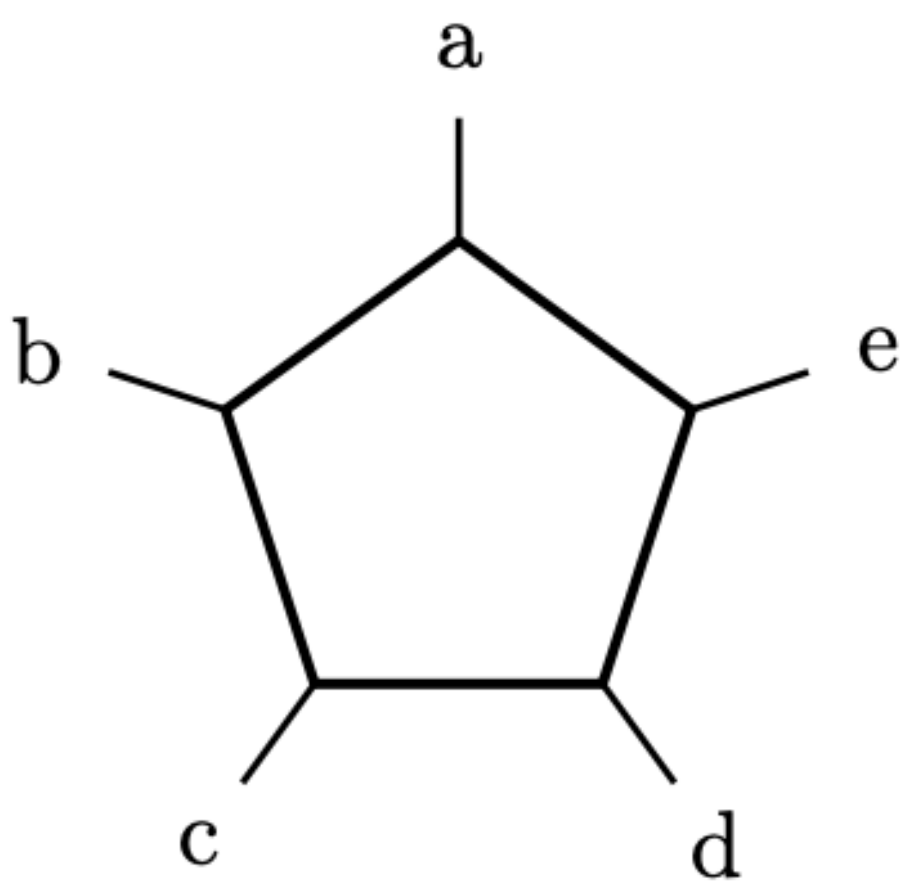

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$$\sum_{c,d,e=0}^{1}\Psi_{a,b,c,d,e}\Psi^{*}_{f,g,c,d,e}=\frac{1}{8}\delta_{a,f}\delta_{b,g}$$

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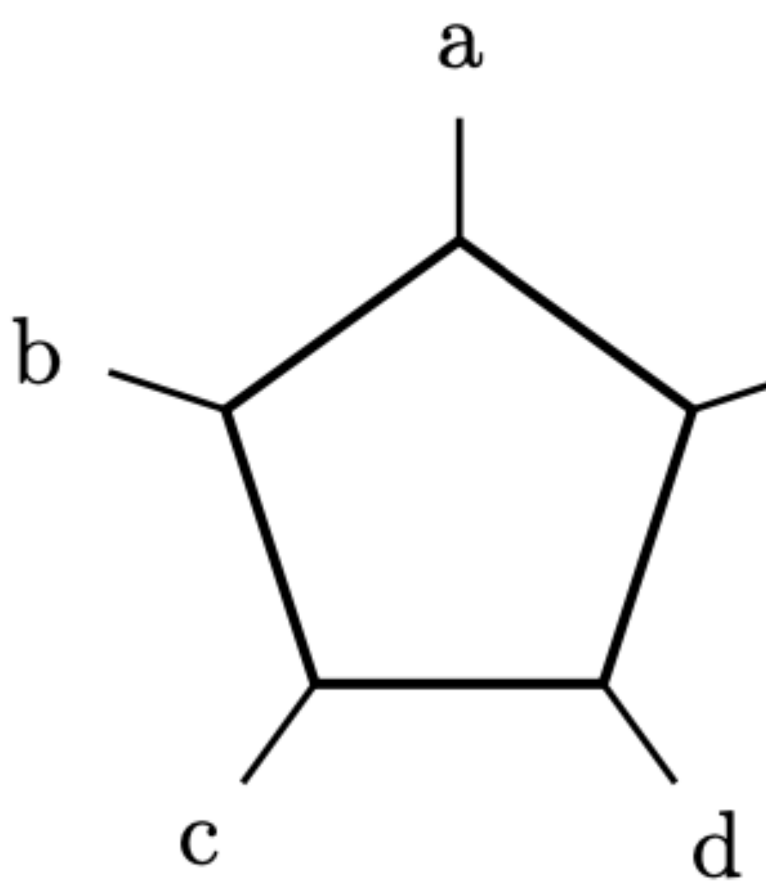

Symmetric configurations:

- 00000
- 10000 + rotations
- 11000 + rotations
- 10100 + rotations
- 11010 + rotations
- 11100 + rotations
- 11110 + rotations
- 11111

Add them up with co-efficients A, B, C, … H!

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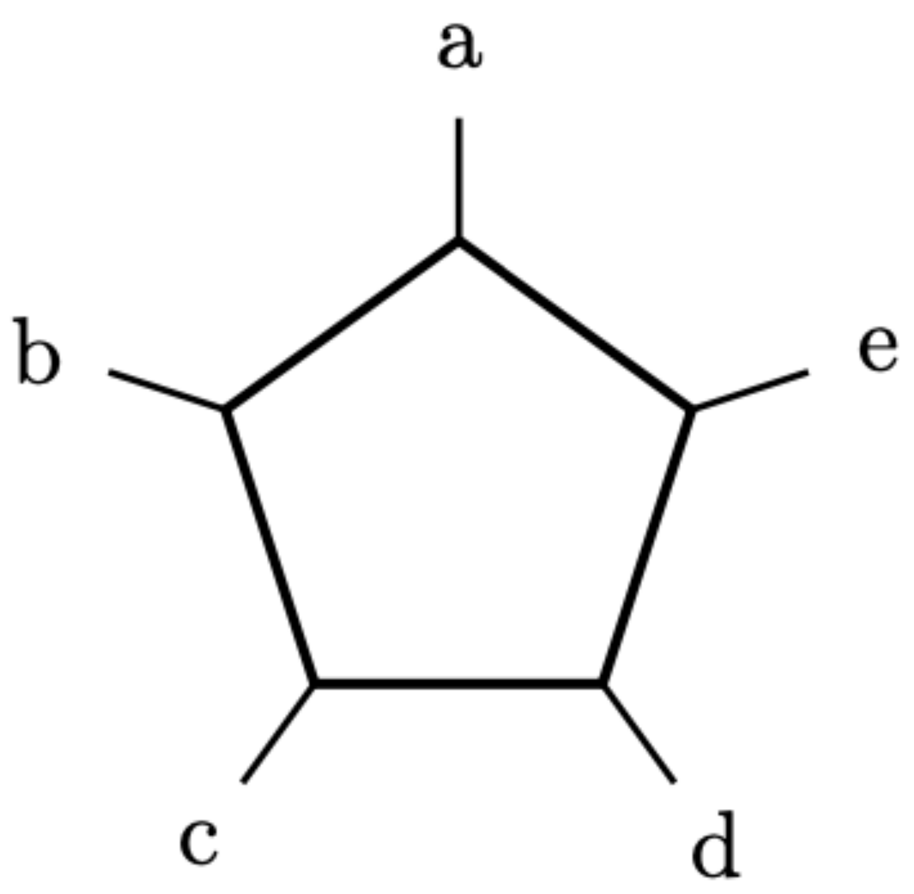

$$|\Psi> = A|00000> + B(|10000> + rot.) + C(|11000> + rot.) + D(|10100> + rot.) +$$
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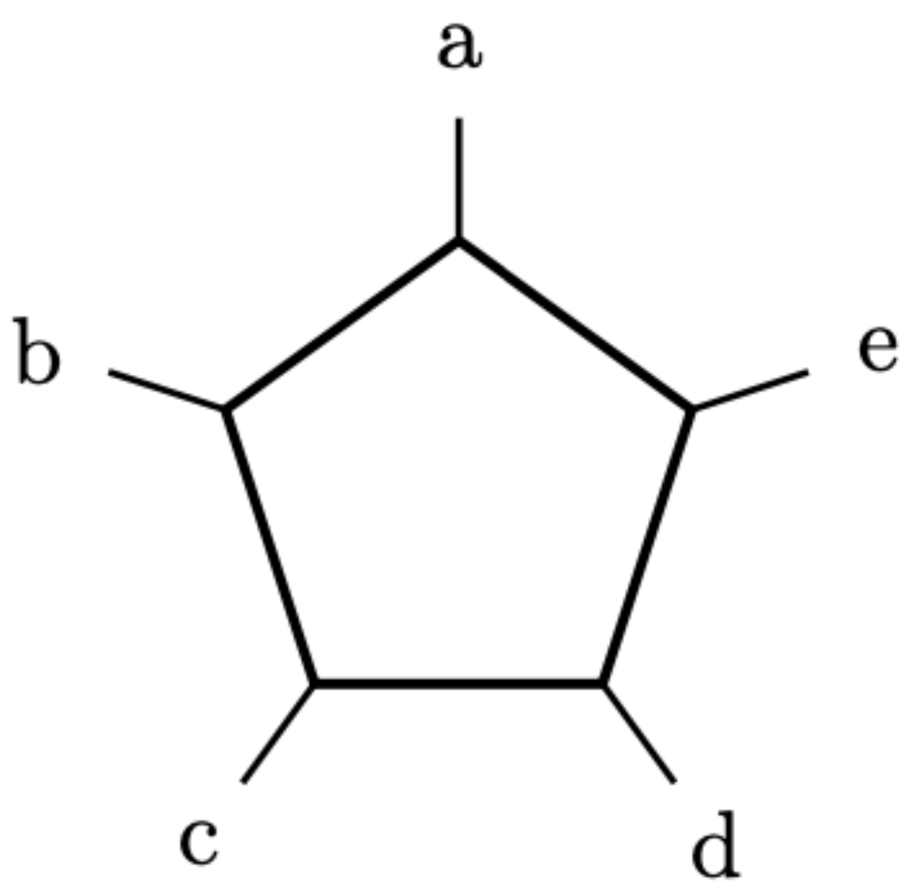

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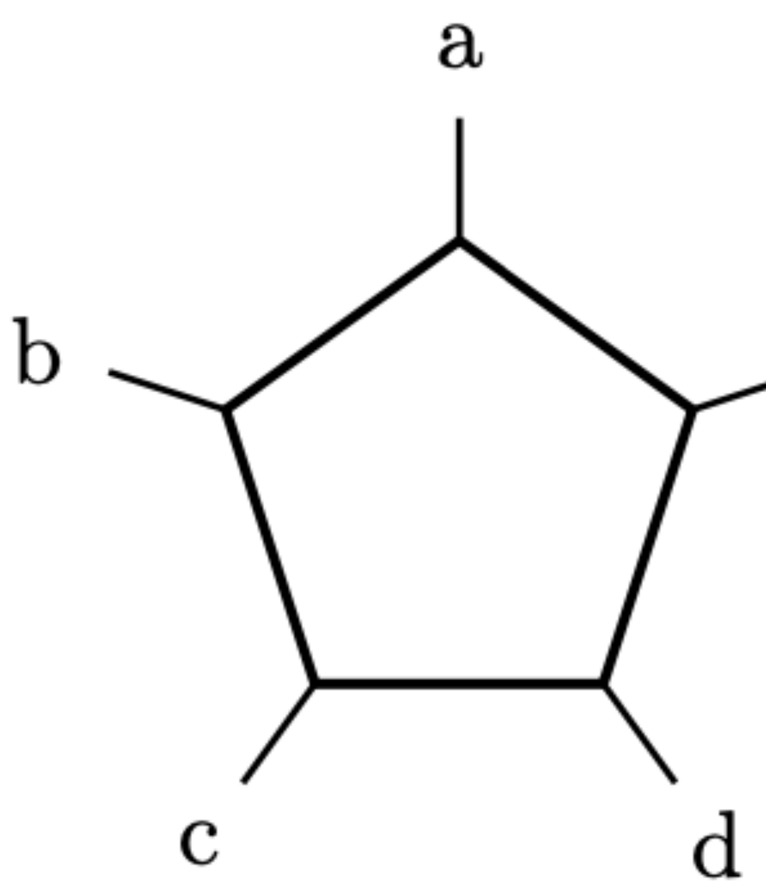

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Assuming real co-efficients:

$$1 = A^2 + 3B^2 + 2C^2 + D^2 + F^2$$
$$1 = B^2 + C^2 + 2D^2 + 2E^2 + F^2 + G^2$$
$$1 = C^2 + E^2 + 2F^2 + 3G^2 + H^2$$
$$0 = AB + BC + 2BD + CE + CF + DE + FG$$
$$0 = B^2 + 2CD + D^2 + E^2 + 2EF + G^2$$
$$0 = AC + BE + 2BF + 2CG + DG + FH$$
$$0 = BC + CF + DE + DF + 2EG + FG + GH$$

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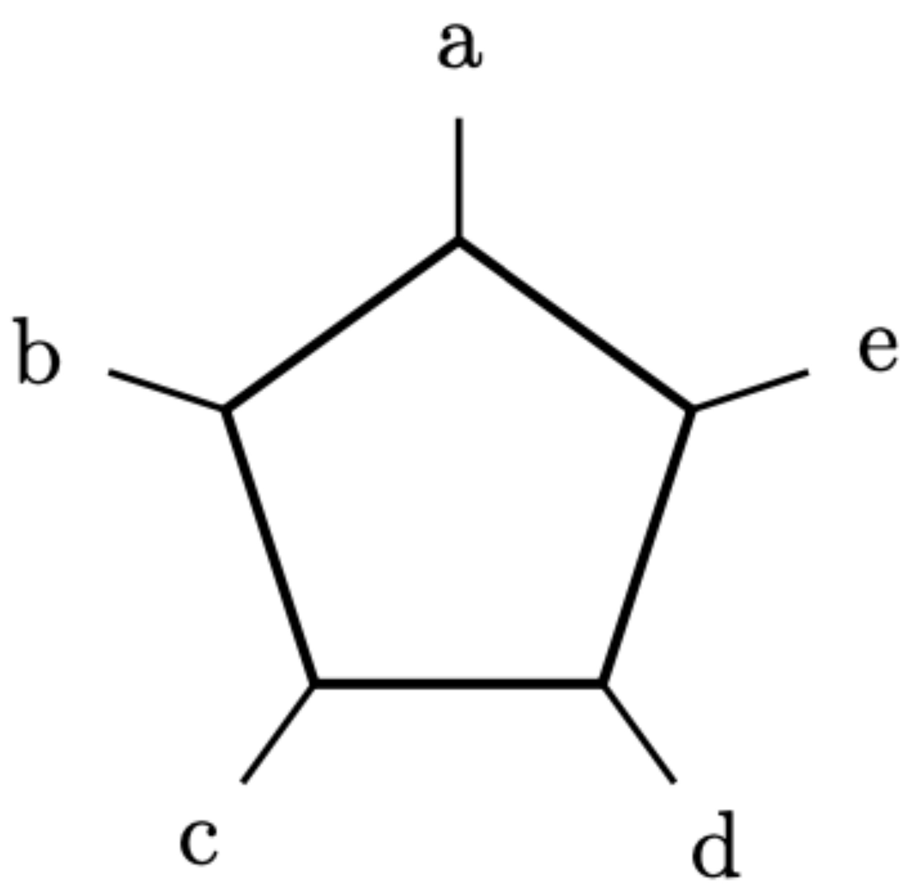

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Family of AME's:

$$A = -C = D = -G = \frac{1}{2}\cos(\alpha)$$
$$B = -E = F = -H = \frac{1}{2}\sin(\alpha)$$

N Ramadas, A. Lakshminarayan, 2025, J. Phys. A. 58 (12), 125301

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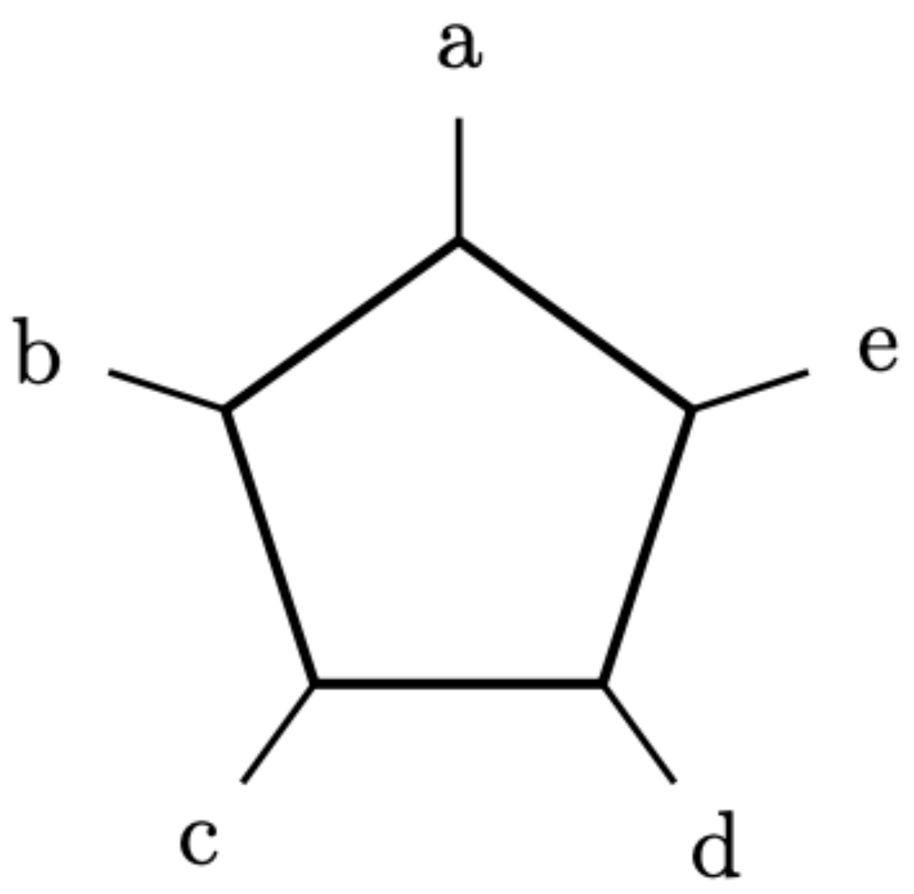
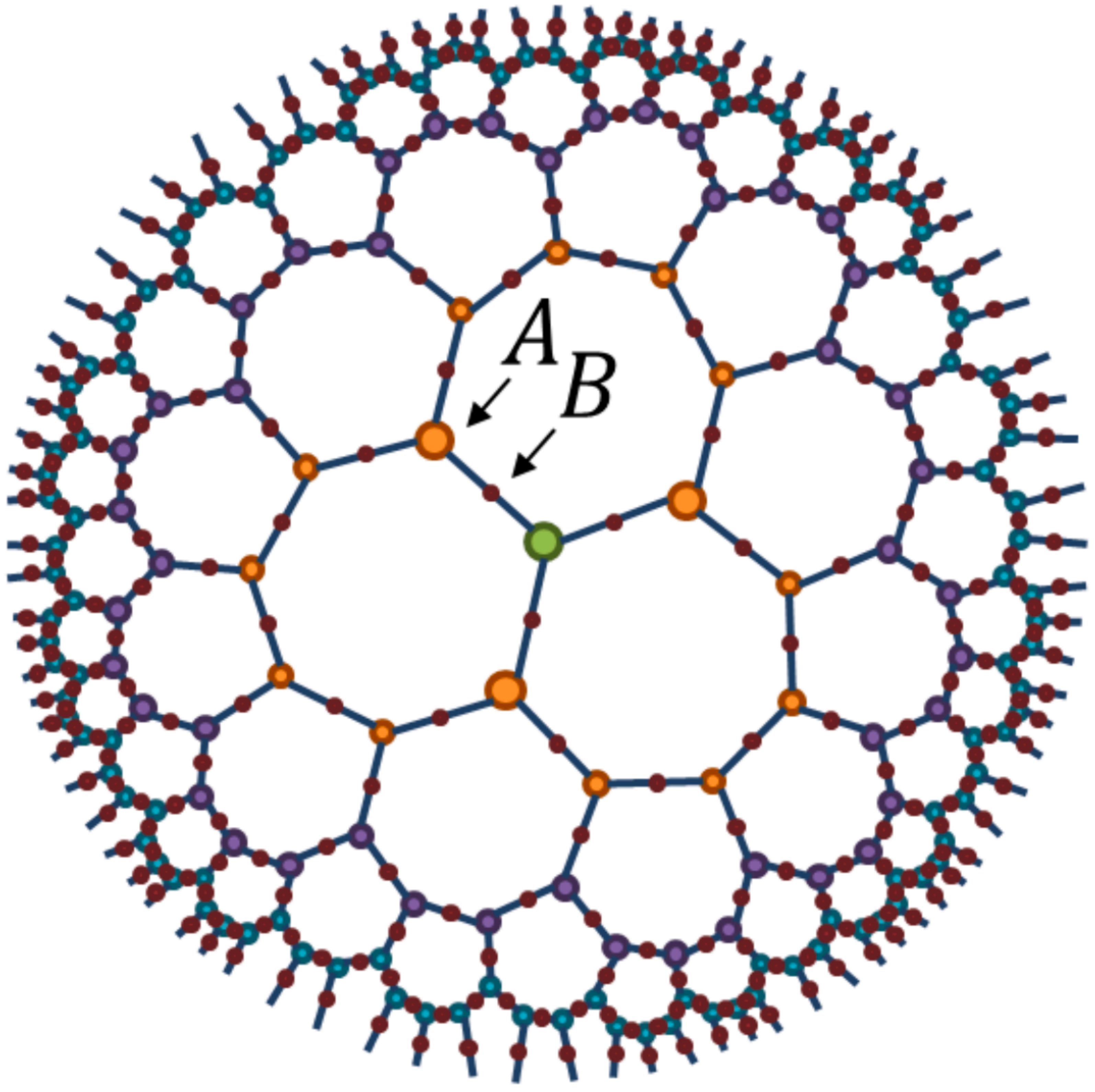


$$A = \frac{1}{4}\left(\sqrt{10\sqrt{5} - 22} + 3\right)$$
$$B = -\frac{1}{4}\left(\sqrt{5} - 2\right)$$
$$C = \frac{1}{4}\left(-(\sqrt{5} - 2) + 2\sqrt{\sqrt{5} - 2}\right)$$
$$D = \frac{1}{4}\left(1 - \sqrt{2\left(\sqrt{5} - 1\right)}\right)$$
$$E = \frac{1}{4}\left(-1 - \sqrt{2\left(\sqrt{5} - 1\right)}\right)$$
$$F = \frac{1}{4}\left(\sqrt{5} - 2 + 2\sqrt{\sqrt{5} - 2}\right)$$
$$G = -B$$
$$H = \frac{1}{4}\left(\sqrt{10\sqrt{5} - 22} - 3\right)$$

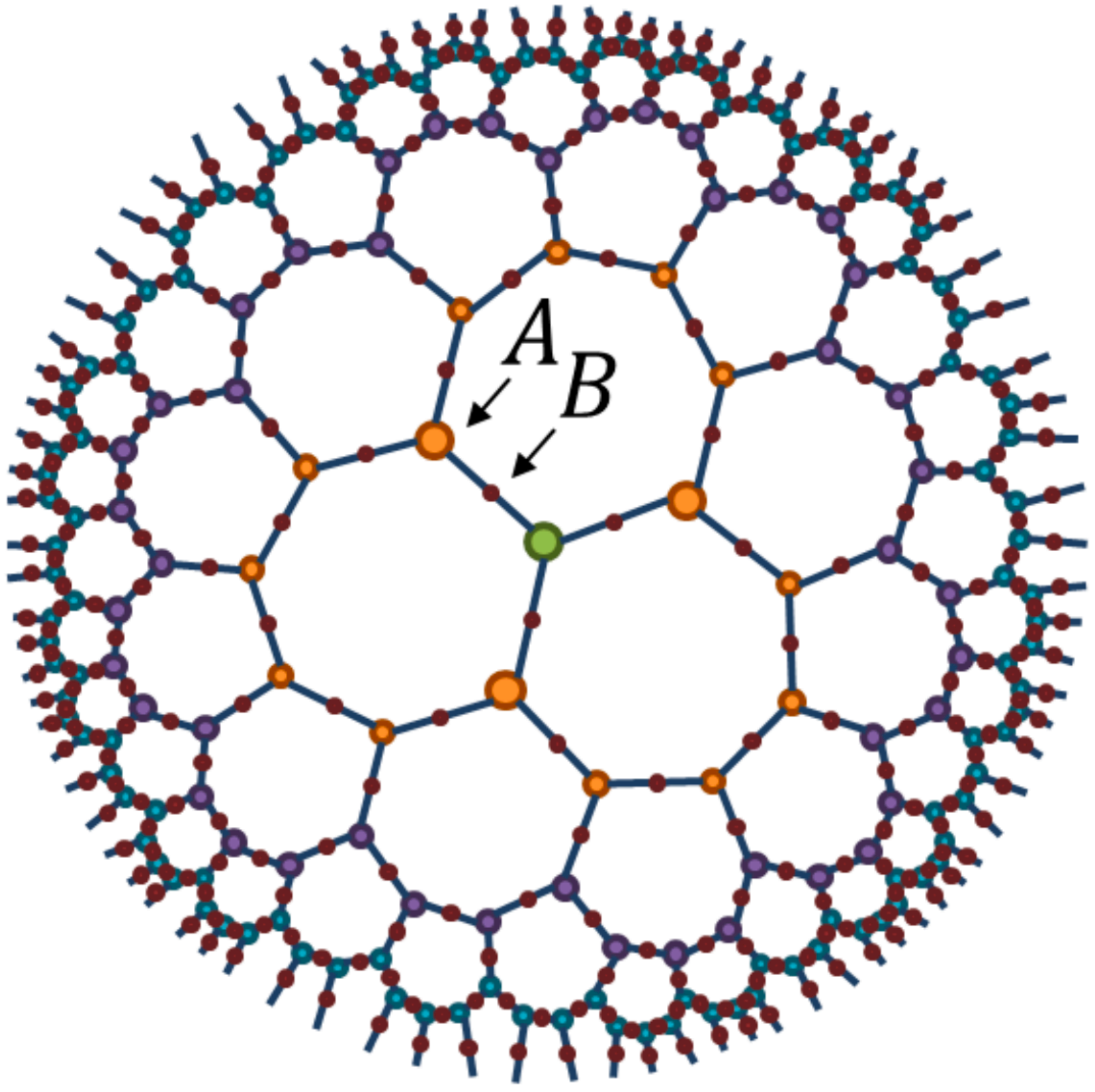


Evenbly (2017)

Lattice models for holography

(=AdS/CFT correspondence)

Holographic error correction codes



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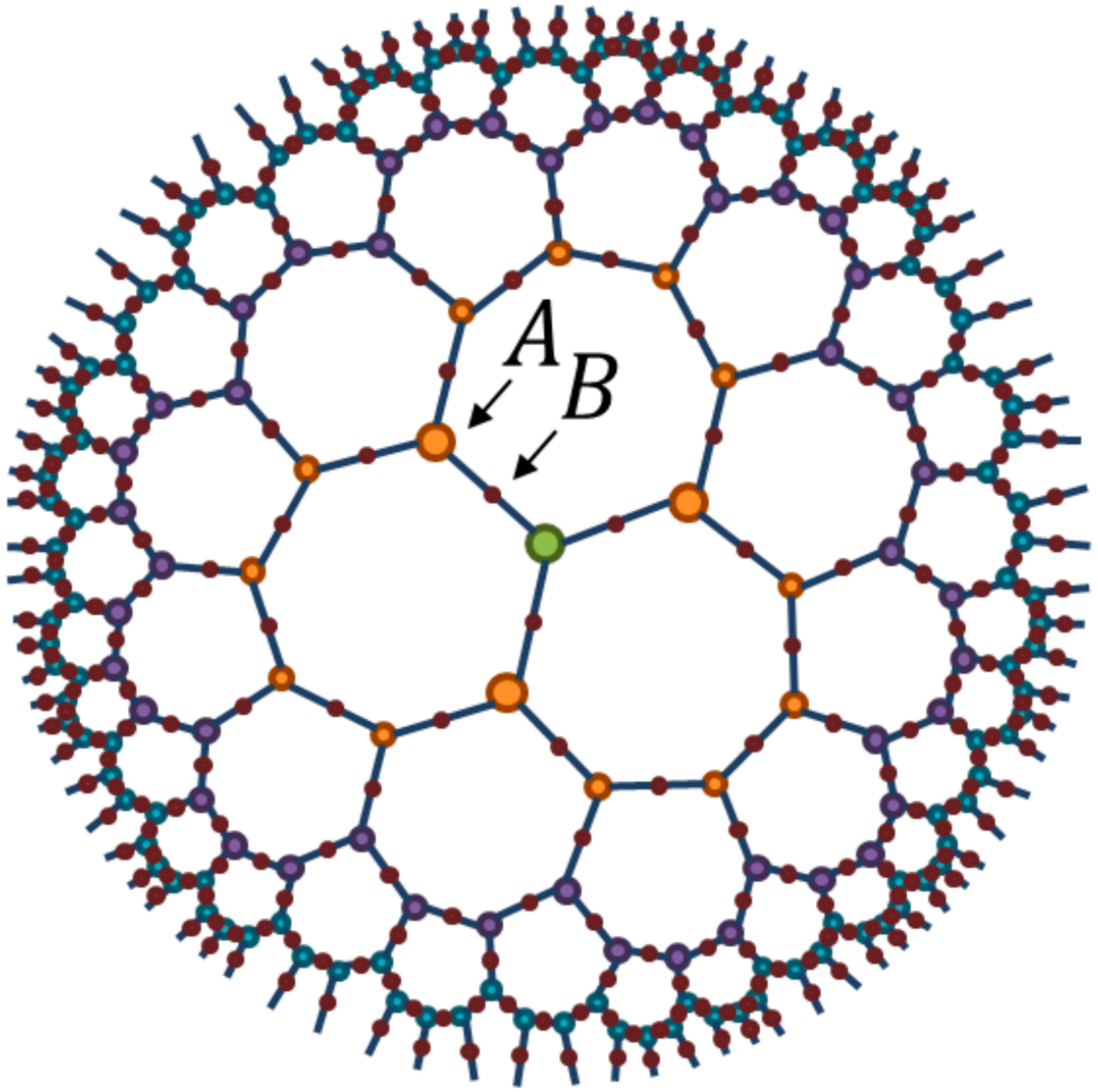
HaPPY code *(Pastawski, Yoshida, Harlow, Preskill, 2015)*



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Perfect tensors —> Trivial correlation functions

Thank you for the attention!

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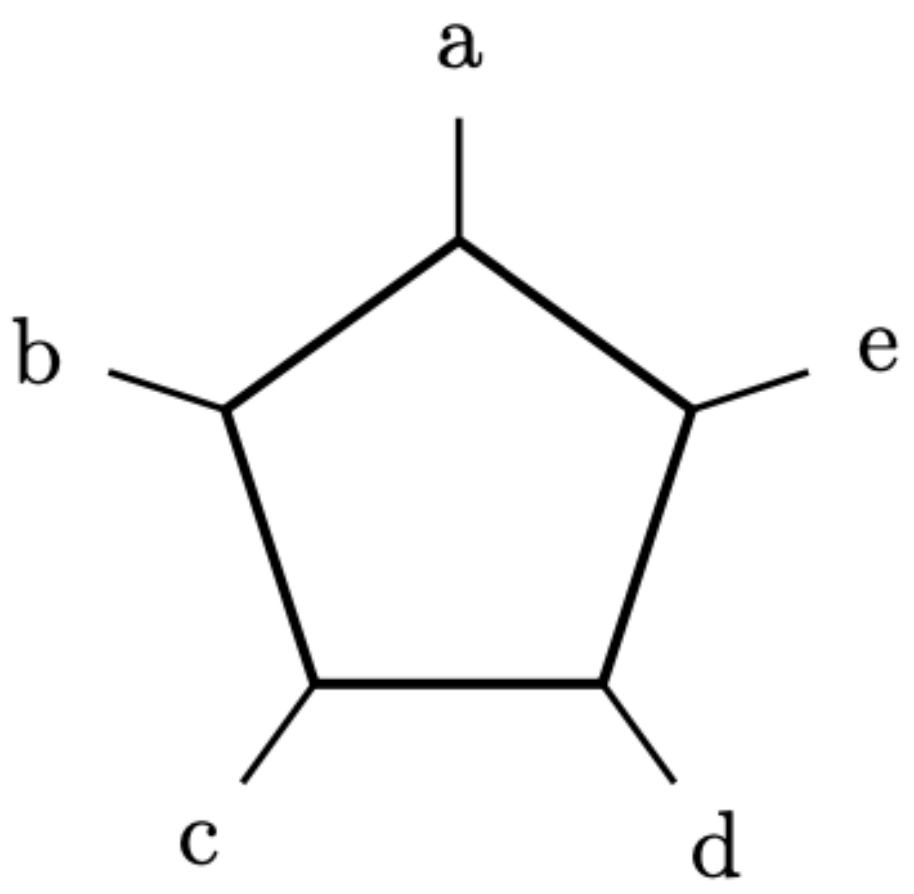


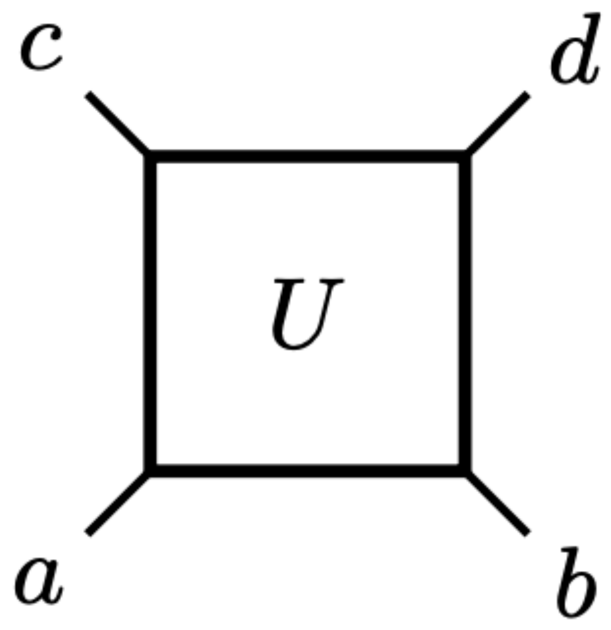
Rotational freedom with

$$Y(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$

Dual unitary operators

= states of 4 qudits, with local dimension D

Dual unitary operators



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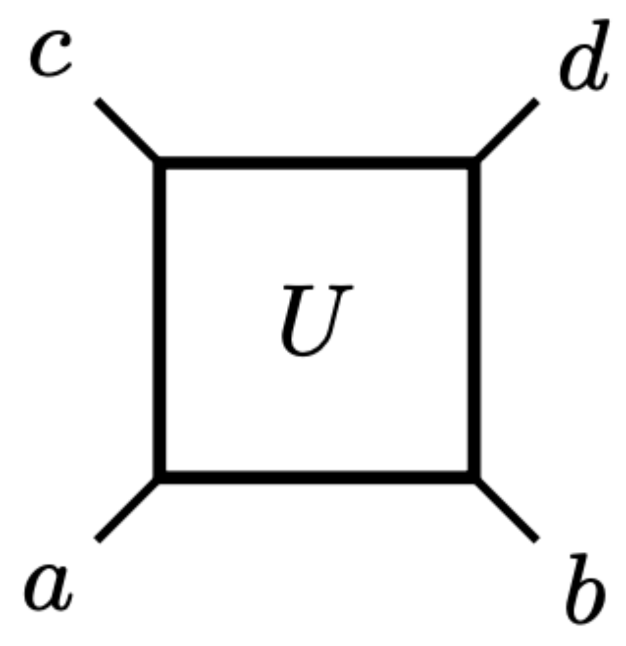

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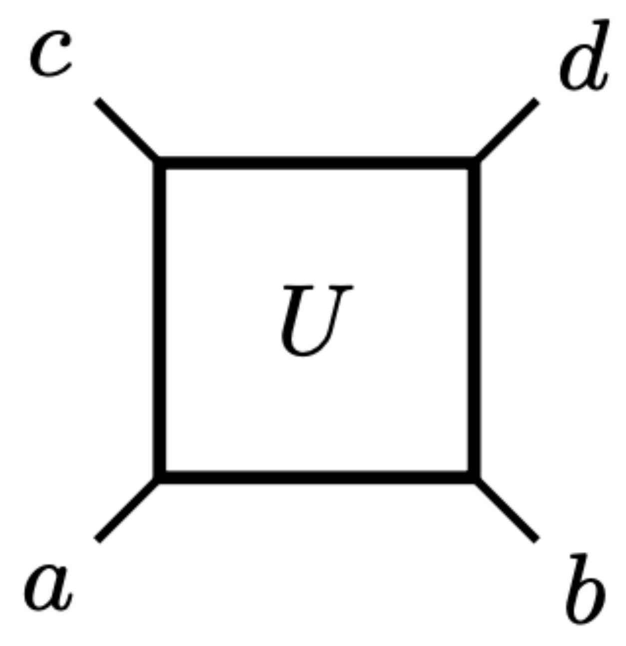

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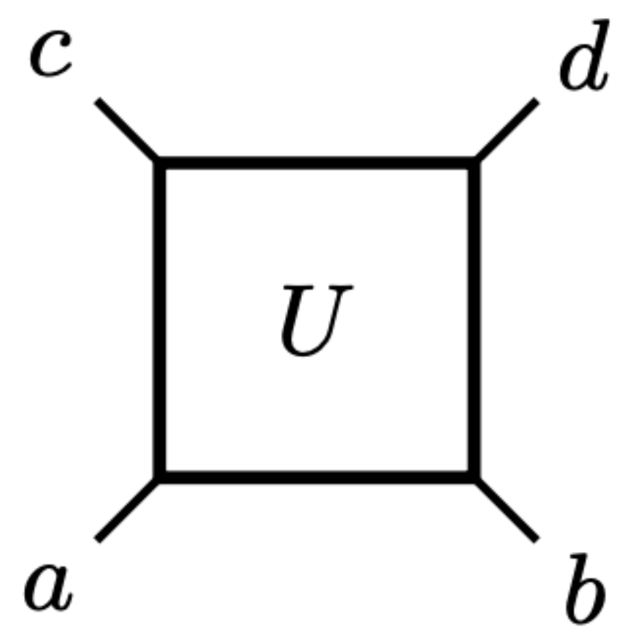

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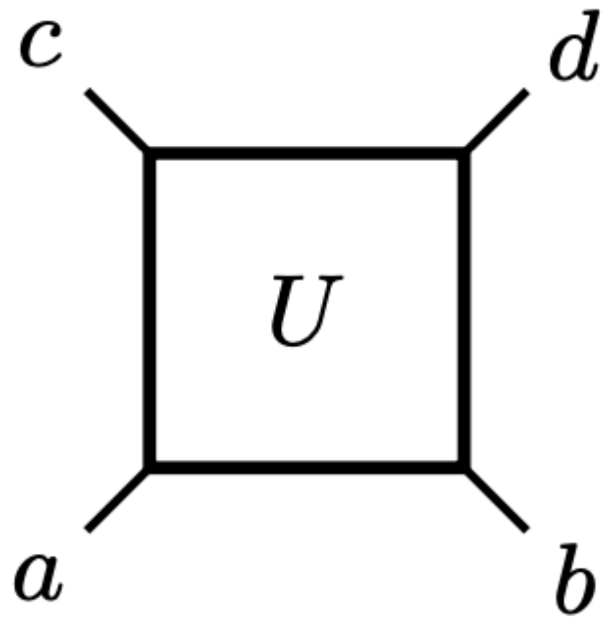

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