Signed groups and Connections to Hadamard (etc) matrices

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Abstract

Independently—indeed literally poles apart geographically—during the late 1980s, two seemingly very different algebraic ideas, signed groups (R. Craigen) and cocyclic index functions (W. de Launey), crept into the study of Hadamard (etc) matrices and designs. Despite their apparent conceptual-level variance, both ideas arise from largely the same underlying mathematical nuts and bolts.

Such convergence seems to indicate an idea whose time has come ... even more so if N. Ito's independent introduction, only slightly later, of Hadamard groups—utilizing that same machinery in yet another way—is taken into account. Further, a substantial body of work has resulted ... from both starting points. Not, as one might suppose, parallel workings of the same theory but at least two quite different bodies of work, diverging both in theoretical impact and in character, apparently illustrating how different formulations of the same machinery may achieve essentially different ends.

Briefly, a **signed group** is a group with a distinguished central element of order 2, denoted multiplicatively as -1. The relationship of signed groups to groups may be regarded as analogous to the relationship of rooted trees to trees.

The impact of cocyclic development of designs, expertly worked out by de Launey, Flannery, Horadam and others is now "well known"; Hadamard groups and signed groups less so. My goal in this talk is to familiarize my audience with the latter, survey the greatest hits (so far) of signed groups and to peel back some more obscure but useful connections.