A generalisation of bent vectors for Butson Hadamard matrices

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Abstract

An $n \times n$ complex matrix M with entries in the k^{th} roots of unity which satisfies $MM^* = nI_n$, where M^* denotes the conjugate transpose of M, is called a Butson Hadamard matrix. While a matrix with entries in the k^{th} roots typically does not have an eigenvector with entries in the same set, such vectors and their generalisations turn out to have multiple applications. In this talk, an M-bent vector is a column vector $\mathbf{x}$ satisfying $M\mathbf{x} = \sqrt{n}\mathbf{y}$ where $\mathbf{x}$ and $\mathbf{y}$ have entries in the k^{th} roots of unity. In particular we study the special case where $\mathbf{y} = \lambda\overline{\mathbf{x}}$, where $\overline{\mathbf{x}}$ is obtained from $\mathbf{x}$ by replacing the entries with their complex conjugate, and λ is a complex number of modulus 1. Such a vector is called a conjugate self-dual bent vector for M.

We will discuss some techniques from algebraic number theory, used to prove some order conditions and non-existence results for self-dual and conjugate self-dual bent vectors. On the existence side, we give examples of many matrices admitting bent vectors using tensor constructions and Bush-type matrices. We conclude with an application to the covering radius of certain non-linear codes generalising the Reed Muller codes.