A construction of Hadamard cubes from association schemes on triples

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Abstract

A d-dimensional Hadamard matrix of order n is a d-dimensional matrix H of order n with entries $1, -1$ such that for any j and distinct a, b,

$$\sum_{1 \leq x_1, \ldots, \hat{x_j}, \ldots, x_d \leq n} H(x_1, \ldots, a, \ldots, x_d) H(x_1, \ldots, b, \ldots, x_d) = n^{d-1} \delta_{ab}.$$

A three-dimensional Hadamard matrix is said to be a Hadamard cube. In [2], Krčadinac, Pavčević, and Tabak used finite fields to construct Hadamard cubes of order $p+1$ where p is a prime power.

Constructions of 2-dimensional Hadamard matrix vary widely. Goethals and Seidel [1] in 1970 showed that a regular symmetric Hadamard matrix with constant diagonal entries is constructed as a linear combination of the identity matrix, and adjacency matrices of a strongly regular graph with certain parameters and its complement. Note that these three $(0, 1)$-matrices define an association scheme.

In this talk, we provide a construction of Hadamard cubes as a linear combination of adjacency matrices of association schemes on triples with certain parameters [3]. Moreover, we will give a construction of association schemes on triples with the desired parameters from any conference matrix. In particular, we prove the following: Let n be the order of a conference matrix. Then there exists a Hadamard cube of order $n+1$.

References

[1] J. M. Goethals, J. J. Seidel, Strongly regular graphs derived from combinatorial designs. *Canad. J. Math.* **22**: 597–614, (1970).

[2] V. Krčadinac, M. O. Pavčević, K. Tabak, Three-dimensional Hadamard matrices of Paley type. *Finite Fields and Their Applications*, **92**: 102306, (2024).

[3] D.M. Mesner, P. Bhattacharya, Association schemes on triples and a ternary algebra, *J. Combin. Theory Ser. A*, **55**: 204–234, (1990).