Mutually Unbiased Bases For Continuous Variables

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(Joint work with Adam Beales)

Abstract

Mutually unbiased bases [1, 2] emerge naturally in infinite-dimensional Hilbert spaces [3] when discussing quantum systems with position and momentum observables, known as (pairs of) continuous variables. We describe similarities and differences between mutually unbiased bases—and, hence Hadamard matrices—arising in finite- and infinite-dimensional Hilbert spaces, respectively. The maximal number of mutually unbiased bases for continuous variables (with basis-independent overlaps) is not known even in the simplest case of a single pair of canonically conjugate observables, corresponding to $N = 1$. For this case, we point out a link between the known triple of mutually unbiased bases and the number of equiangular lines in $\mathbb{R}^2$, the "phase space" of the system. We conjecture that this relation extends to more pairs of continuous variables, i.e. to $N > 1$: one can construct at least as many mutually unbiases bases as there are equiangular lines in $\mathbb{R}^{2N}$.

References

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